LESSON MAKER PRO

Twice the Angle

Build double-angle identities from angle sums.

HIGH SCHOOL · PC.F.H.37.b

PC.F.H.37.b "Use the angle sum formulas for sine, cosine, and tangent to derive the double angle formulas."

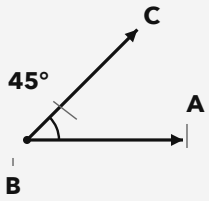
NAME _____ DATE _____

START WITH A SUM

Use sin(A+B)=sin A cos B+cos A sin B and cos(A+B)=cos A cos B−sin A sin B. Also use tan(A+B)=(tan A+tan B)/(1−tan A tan B), where defined. Set A and B equal to build double-angle identities.

WORKED EXAMPLE

In the figure, x is angle ABC. Use the sine sum formula to find sin(2x).



1. sin(2x)=sin(x+x)
2. sin(x+x)=sin x cos x+cos x sin x=2sin x cos x
3. sin x=cos x=√2/2
4. sin(2x)=2(√2/2)(√2/2)=1

Answer: 1

PRACTICE 4 problems

1 Set A=B=x in the cosine sum formula. Derive cos(2x).

ANSWER _____

2 Set A=B=x in the tangent sum formula. Derive tan(2x).

ANSWER _____

3 Use $\sin^2 x+\cos^2 x=1$ to write cos(2x) using only sin x.

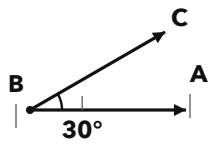
ANSWER _____

4 If sin x=3/5 and cos x=4/5, use the sine sum formula to find sin(2x).

ANSWER _____

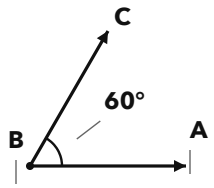
APPLY IT Show your work

1 A video editor rotates a logo by the angle shown, then repeats that rotation. Use the cosine sum formula to find the cosine of the total turn.



ANSWER _____

2 In a game, a camera turns by the angle shown, then turns that much again. Use the tangent sum formula to find the tangent of the total turn.



ANSWER _____

CHALLENGE One step up

Use double-angle identities twice to derive a formula for sin(4x) using only sin x and cos x. Show your steps.

WORK SPACE AND ANSWER

ANSWER KEY

Twice the Angle

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Set A=B=x in the cosine sum formula. Derive cos(2x).	**$\cos(2x)=\cos^2x-\sin^2x$**
2	Set A=B=x in the tangent sum formula. Derive tan(2x).	**$\tan(2x)=2\tan x/(1-\tan^2x)$, where defined**
3	Use $\sin^2x+\cos^2x=1$ to write cos(2x) using only sin x.	**$\cos(2x)=1-2\sin^2x$**
4	If sin x=3/5 and cos x=4/5, use the sine sum formula to find sin(2x).	**24/25**

PART 2 · APPLY IT

1. **1/2** The total turn is 30°+30°. $\cos(30°+30°)=(\sqrt{3}/2)^2-(1/2)^2=1/2$.
2. **$-\sqrt{3}$** The total turn is 60°+60°. $\tan(60°+60°)=(\sqrt{3}+\sqrt{3})/(1-3)=-\sqrt{3}$.

CHALLENGE

$\sin(4x)=2\sin(2x)\cos(2x)=2(2\sin x\cos x)(\cos^2x-\sin^2x)=4\sin x\cos x(\cos^2x-\sin^2x)$.

Accept equivalent forms. For tangent identities, the denominator must be nonzero.