LESSON MAKER PRO

Double Angle Detective

Use familiar identities to prove new ones.

HIGH SCHOOL · PC.F.H.37.c

PC.F.H.37.c "Use the Pythagorean and double angle identities to prove other simple identities."

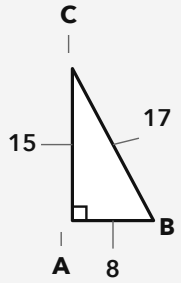
NAME _____ DATE _____

BUILD AN IDENTITY

The Pythagorean identity is $\sin^2 x + \cos^2 x = 1$. The double-angle identities are $\sin(2x) = 2\sin x \cos x$ and $\cos(2x) = \cos^2 x - \sin^2 x$. To prove an identity, replace an expression with an equal one, then simplify until both sides match.

WORKED EXAMPLE

In the figure, x is angle B. Use a double-angle identity to find sin(2x).

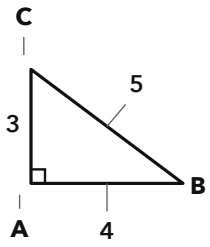


1. At angle B, $\sin x = 15/17$ and $\cos x = 8/17$.
2. Use $\sin(2x) = 2\sin x \cos x$.
3. $\sin(2x) = 2(15/17)(8/17) = 240/289$.

Answer: sin(2x) = 240/289

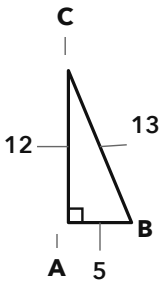
PRACTICE 6 problems

1 In the figure, x is angle B. Use $\cos(2x) = 1 - 2\sin^2 x$ to find $\cos(2x)$.



ANSWER _____

2 In the figure, x is angle B. Use $\cos(2x) = 2\cos^2 x - 1$ to find $\cos(2x)$.



ANSWER _____

3 Starting with $\cos(2x) = \cos^2 x - \sin^2 x$, prove $\cos(2x) = 1 - 2\sin^2 x$.

ANSWER _____

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4 Starting with $\cos(2x) = \cos^2 x - \sin^2 x$, prove $\cos(2x) = 2\cos^2 x - 1$.

ANSWER _____

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5 Prove $(\sin x + \cos x)^2 = 1 + \sin(2x)$.

ANSWER _____

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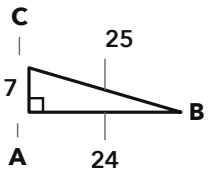
6 Prove $1 - \cos^2(2x) = 4\sin^2 x \cos^2 x$.

ANSWER _____

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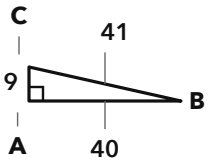
APPLY IT Show your work

1 A video creator tilts a phone camera by angle x in the figure. An editing tool uses cos(2x) to set an effect. Find the exact value of cos(2x).



ANSWER _____

2 A skateboard ramp has the profile shown, with x as its angle at B. A motion app uses sin(2x) as a movement factor. Find its exact value.



ANSWER _____

CHALLENGE One step up

For $\cos x \neq \sin x$, prove $(1 + \sin(2x))/(1 - \sin(2x)) = ((\sin x + \cos x)/(\cos x - \sin x))^2$. Show how both the numerator and denominator can be rewritten.

WORK SPACE AND ANSWER

Double Angle Detective

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	In the figure, x is angle B. Use $\cos(2x) = 1 - 2\sin^2 x$ to find $\cos(2x)$.	**7/25**
2	In the figure, x is angle B. Use $\cos(2x) = 2\cos^2 x - 1$ to find $\cos(2x)$.	**-119/169**
3	Starting with $\cos(2x) = \cos^2 x - \sin^2 x$, prove $\cos(2x) = 1 - 2\sin^2 x$.	**$\cos(2x) = (1 - \sin^2 x) - \sin^2 x = 1 - 2\sin^2 x$.**
4	Starting with $\cos(2x) = \cos^2 x - \sin^2 x$, prove $\cos(2x) = 2\cos^2 x - 1$.	**$\cos(2x) = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$.**
5	Prove $(\sin x + \cos x)^2 = 1 + \sin(2x)$.	**$(\sin x + \cos x)^2 = \sin^2 x + 2\sin x \cos x + \cos^2 x = 1 + \sin(2x)$.**
6	Prove $1 - \cos^2(2x) = 4\sin^2 x \cos^2 x$.	**$1 - \cos^2(2x) = \sin^2(2x) = (2\sin x \cos x)^2 = 4\sin^2 x \cos^2 x$.**

PART 2 · APPLY IT

1. **527/625** Use x as angle B. Since $\cos x = 24/25$, $\cos(2x) = 2(24/25)^2 - 1 = 527/625$.
2. **720/1681** At angle B, $\sin x = 9/41$ and $\cos x = 40/41$. Thus $\sin(2x) = 2(9/41)(40/41) = 720/1681$.

CHALLENGE

$1 + \sin(2x) = \sin^2 x + 2\sin x \cos x + \cos^2 x = (\sin x + \cos x)^2$. Similarly, $1 - \sin(2x) = \cos^2 x - 2\sin x \cos x + \sin^2 x = (\cos x - \sin x)^2$. Substitute these expressions into the fraction.

Accept equivalent identity proofs that use valid substitutions. In the triangle figures, angle A is the right angle and the side opposite it is the hypotenuse.