LESSON MAKER PRO

Double Angle Moves

Use one angle to find another.

HIGH SCHOOL · PC.F.H.37

PC.F.H.37 "Use trigonometric identities to solve problems. a. Use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ to derive the other forms of the identity. b. Use the angle sum formulas for sine, cosine..."

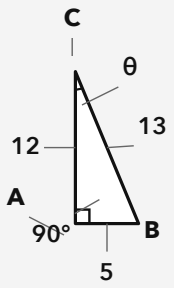
NAME _____ **DATE** _____

THE BIG IDEA

Use $\sin^2\theta + \cos^2\theta = 1$ to find a missing sine or cosine. The angle sum formulas give $\sin(2\theta) = 2\sin\theta\cos\theta$, $\cos(2\theta) = \cos^2\theta - \sin^2\theta$, and $\tan(2\theta) = 2\tan\theta/(1 - \tan^2\theta)$ when defined.

WORKED EXAMPLE

Find $\sin(2\theta)$ for the angle shown.



1. $\sin\theta = 5/13$.
2. $\cos\theta = 12/13$.
3. $\sin(2\theta) = 2(5/13)(12/13) = 120/169$.

Answer: 120/169

PRACTICE 4 problems

1 If $\sin\theta = 3/5$ and θ is acute, find $\sin(2\theta)$.

ANSWER _____

2 If $\cos\theta = 8/17$ and θ is acute, find $\cos(2\theta)$.

ANSWER _____

3 Start with $\tan(x + x)$ and derive a formula for $\tan(2x)$.

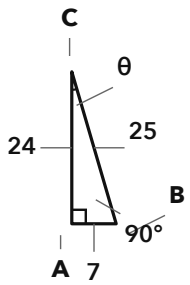
ANSWER _____

4 Use the cosine sum formula and the Pythagorean identity to show $\cos(2x) = 1 - 2\sin^2 x$.

ANSWER _____

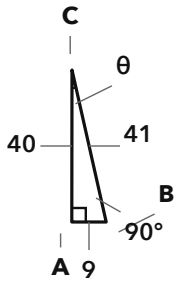
APPLY IT Show your work

1 A skateboard game doubles the ramp angle θ shown to set a trick angle. Find the sine of the trick angle.



ANSWER _____

2 A drone simulator doubles the climb angle θ shown. Find the cosine of the new angle.



ANSWER _____

CHALLENGE One step up

Prove sin(2x)/(1 + cos(2x)) = tan x wherever both sides are defined.

WORK SPACE AND ANSWER

ANSWER KEY

Double Angle Moves

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	If $\sin\theta = 3/5$ and θ is acute, find $\sin(2\theta)$.	**24/25**
2	If $\cos\theta = 8/17$ and θ is acute, find $\cos(2\theta)$.	**−161/289**
3	Start with $\tan(x + x)$ and derive a formula for $\tan(2x)$.	**$\tan(2x) = (\tan x + \tan x)/(1 - \tan x \tan x) = 2\tan x/(1 - \tan^2 x)$, when defined.**
4	Use the cosine sum formula and the Pythagorean identity to show $\cos(2x) = 1 - 2\sin^2 x$.	**$\cos(2x) = \cos^2 x - \sin^2 x = (1 - \sin^2 x) - \sin^2 x = 1 - 2\sin^2 x$.**

PART 2 · APPLY IT

1. **336/625** The figure gives $\sin\theta = 7/25$ and $\cos\theta = 24/25$. Thus $\sin(2\theta) = 2(7/25)(24/25) = 336/625$.
2. **1519/1681** The figure gives $\cos\theta = 40/41$ and $\sin\theta = 9/41$. Thus $\cos(2\theta) = (40/41)^2 - (9/41)^2 = 1519/1681$.

CHALLENGE

$\sin(2x)/(1 + \cos(2x)) = 2\sin x \cos x/(1 + \cos^2 x - \sin^2 x) = 2\sin x \cos x/(2\cos^2 x) = \tan x$.

Accept equivalent exact fractions and valid alternate identity proofs. For the tangent formula, the stated restriction means both $\tan x$ and $\tan(2x)$ must be defined.