LESSON MAKER PRO

Zero Detective

Find the zeros a graph shows and the ones it cannot.

HIGH SCHOOL · PC.NQ.C.6

PC.NQ.C.6 "Analyze possible zeros for a polynomial function over the complex numbers by applying the Fundamental Theorem of Algebra, using a graph of the function, or factoring with algebraic identities."

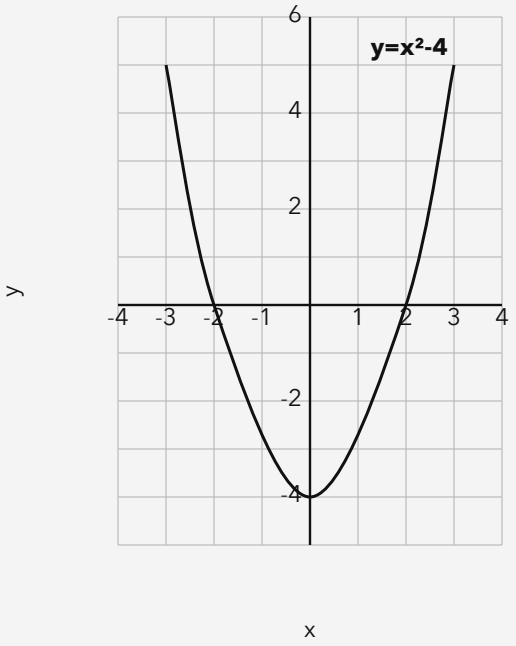
NAME _____ DATE _____

COUNT EVERY ZERO

A polynomial has as many complex zeros as its degree, counting repeated zeros. A graph shows real zeros at its x-intercepts, but it cannot show nonreal zeros. Factoring can reveal the remaining zeros, and nonreal zeros of a polynomial with real coefficients come in conjugate pairs.

WORKED EXAMPLE

The figure shows $y=x^2-4$. Find all its complex zeros.

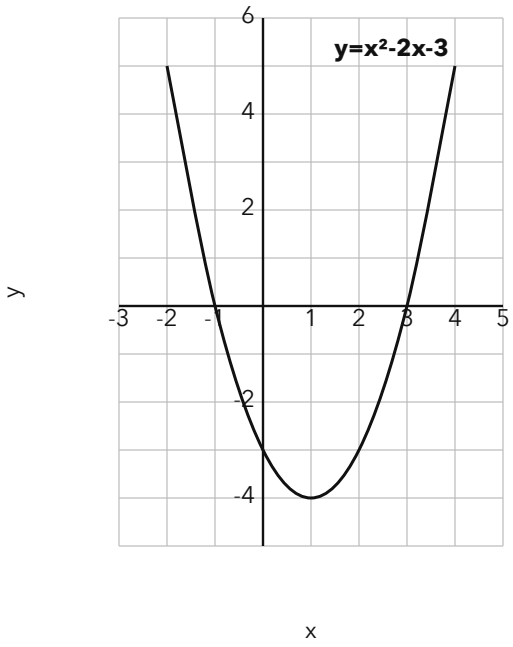


1. Read the x-intercepts: x=-2 and x=2.
2. The polynomial has degree 2, so it has 2 complex zeros counting repeats.
3. Both zeros appear on the graph, so no nonreal zeros remain.

Answer: -2 and 2

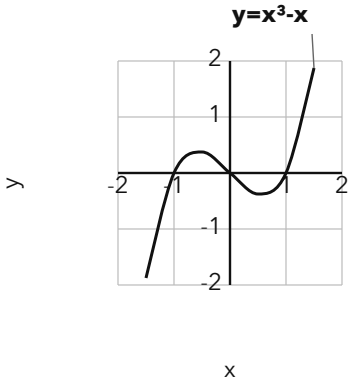
PRACTICE 6 problems

1 The figure shows $y=x^2-2x-3$. List all its complex zeros.



ANSWER _____

2 The figure shows $y=x^3-x$. List its real zeros. How many nonreal zeros does it have?



ANSWER _____

_____

3 Find all complex zeros of x^2+25.

ANSWER _____

4 Find all complex zeros of $x^2-8x+16$. State the multiplicity of each zero.

ANSWER _____

_____

5 Factor x^3-9x, then list all its complex zeros.

ANSWER _____

_____

6 Find all complex zeros of x^4+10x^2+9.

ANSWER _____

APPLY IT Show your work

1 A game designer models a character's height above a platform with $h(x)=-x^2+6x-5$, where x is seconds after the jump starts and $1<=x<=5$. At what times is the character at platform height? Are there any nonreal zeros?

WORK SPACE

ANSWER _____

2 A video editor models an effect's intensity with $I(x)=x^3+16x$, where x is seconds since the effect begins and $0<=x<=1$. Find all complex zeros. Which zero represents a time when the intensity is zero?

WORK SPACE

ANSWER _____

CHALLENGE One step up

One zero of $x^3-x^2-7x+15$ is $2+i$. Find the other two zeros, and explain how you know you found them all.

WORK SPACE AND ANSWER

Zero Detective

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	The figure shows $y=x^2-2x-3$. List all its complex zeros.	**-1 and 3**
2	The figure shows $y=x^3-x$. List its real zeros. How many nonreal zeros does it have?	**Real zeros: -1, 0, 1. Nonreal zeros: 0.**
3	Find all complex zeros of x^2+25.	**5i and -5i**
4	Find all complex zeros of $x^2-8x+16$. State the multiplicity of each zero.	**4, with multiplicity 2**
5	Factor x^3-9x, then list all its complex zeros.	**x(x-3)(x+3); zeros: -3, 0, 3**
6	Find all complex zeros of x^4+10x^2+9.	**i, -i, 3i, -3i**

PART 2 · APPLY IT

1. **At 1 second and 5 seconds; there are no nonreal zeros.** Factor $h(x)=-(x-1)(x-5)$. Its two real zeros account for both zeros of a quadratic.
2. **The complex zeros are 0, 4i, and -4i. Only x=0 represents a time.** Factor $I(x)=x(x^2+16)$. The nonreal zeros cannot represent times.

CHALLENGE

The other zeros are 2-i and -3. Because the polynomial has real coefficients, 2-i is also a zero. Factoring gives $(x^2-4x+5)(x+3)$, which accounts for all three zeros of a cubic.

Accept zeros in any order. For the challenge, accept an equivalent justification that uses the conjugate pair and the degree of the polynomial.