LESSON MAKER PRO

Where Does It Go?

Use limits to track what happens near a value and at infinity.

HIGH SCHOOL · PC.NQ.D.7.a

PC.NQ.D.7.a "Apply limits of functions at specific values and at infinity in problems involving convergence and divergence."

NAME _____ DATE _____

FINDING A LIMIT

To find a limit as x approaches a number, try substituting that number. If a fraction gives 0/0, simplify it before substituting again. As x approaches infinity, compare the highest powers of x to decide whether the function approaches a value or grows without bound.

WORKED EXAMPLE

Find the limit of $(x^2 - 49)/(x + 7)$ as x approaches -7.

1. Substituting -7 gives 0/0.
2. Factor: $(x^2 - 49)/(x + 7) = (x - 7)(x + 7)/(x + 7)$.
3. For x not equal to -7, cancel x + 7 to get x - 7.
4. As x approaches -7, x - 7 approaches -14.

Answer: -14

PRACTICE 6 problems

1 Find the limit of $x^2 + 3x + 4$ as x approaches -1.

ANSWER _____

2 Find the limit of $(x^2 - 16)/(x - 4)$ as x approaches 4.

ANSWER _____

3 Find the limit of $(5x + 2)/(x + 7)$ as x approaches infinity.

ANSWER _____

4 Find the limit of $9/(2x^2 + 1)$ as x approaches infinity.

ANSWER _____

5 As x approaches infinity, does $(2x^2 + 1)/(x + 3)$ converge or diverge? State what happens to its value.

ANSWER _____

6 Does $1/(x - 2)$ have a limit as x approaches 2? Give its behavior from the left and from the right.

ANSWER _____

APPLY IT Show your work

1 A sticker shop charges $24 for shipping plus $3 per sticker. If you buy x stickers, the average cost per sticker is A(x) = (24 + 3x)/x dollars. Find the limit of A(x) as x approaches infinity. What does the limit mean?

WORK SPACE

ANSWER _____

2 A basketball's height x seconds after a shot is $h(x) = 2 + 8x - 5x^2$ meters. Its average vertical speed from 1 second to x seconds is (h(x) - h(1))/(x - 1). Find the limit of this speed as x approaches 1, and say whether the ball is rising or falling then.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Let $f(x) = (x^2 - 9)/(x^2 - 6x + 9)$. Find what happens to f(x) as x approaches 3 from the left and from the right. Does the two-sided limit at 3 exist? Then find the limit as x approaches infinity.

WORK SPACE AND ANSWER

ANSWER KEY

Where Does It Go?

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the limit of $x^2 + 3x + 4$ as x approaches -1.	**2**
2	Find the limit of $(x^2 - 16)/(x - 4)$ as x approaches 4.	**8**
3	Find the limit of $(5x + 2)/(x + 7)$ as x approaches infinity.	**5**
4	Find the limit of $9/(2x^2 + 1)$ as x approaches infinity.	**0**
5	As x approaches infinity, does $(2x^2 + 1)/(x + 3)$ converge or diverge? State what happens to its value.	**Diverges; its value grows toward +infinity.**
6	Does $1/(x - 2)$ have a limit as x approaches 2? Give its behavior from the left and from the right.	**No. From the left it approaches -infinity; from the right it approaches +infinity.**

PART 2 · APPLY IT

1. **$3 per sticker. As the order grows, the shipping cost per sticker approaches $0.** Rewrite A(x) as 24/x + 3. As x approaches infinity, 24/x approaches 0.
2. **-2 meters per second; the ball is falling.** Since h(1) = 5, factor (h(x) - 5)/(x - 1) to get 3 - 5x for x not equal to 1. Its limit as x approaches 1 is -2.

CHALLENGE

As x approaches 3 from the left, f(x) approaches -infinity. From the right, it approaches +infinity, so the two-sided limit does not exist. As x approaches infinity, f(x) approaches 1.

Accept equivalent descriptions of unbounded growth and one-sided behavior. In the challenge, factoring gives f(x) = (x + 3)/(x - 3) for x not equal to 3.