Orbit Forces

Model how gravity keeps a spacecraft moving in a circle.

P.5 "Develop and use models to analyze the circular motion of objects. a. Use mathematics and free-body diagrams to relate the tangential velocity, the radius of orbit, the centripetal acceleration..."

NAME _____ DATE _____

1 READ AND MARK

Read it twice

YOUR JOB Underline the equations that connect speed, radius, acceleration, and force. Circle each statement that tells you which way to draw an arrow in a model of circular motion.

What Keeps a Spacecraft in Orbit?

1. A spacecraft circling Earth is not moving in a straight line, even if its speed stays steady. At each moment, its velocity points along the path, while its acceleration points toward the center of the circle. For a circle of radius r and tangential speed v, the centripetal acceleration has magnitude $a_c = v^2/r$. Newton's second law gives the required inward net force as $F_{net} = ma_c = mv^2/r$. Speed matters strongly: if v increases while r stays fixed, the needed inward force rises with the square of v. A free-body diagram shows forces, not velocity or acceleration.

2. For a spacecraft in an ideal circular orbit, gravity supplies that inward force. In a free-body diagram of the spacecraft, draw one arrow toward Earth's center and label it gravitational force. Do not add an outward force: the spacecraft's tendency to keep moving straight is not a force. Newton's law of universal gravitation gives $F_g = GMm/r^2$, where M is Earth's mass, m is the spacecraft's mass, and r is the distance between their centers. Use $G = 6.67 \times 10^{-11}\ N \cdot m^2/kg^2$ and $M = 6.0 \times 10^{24}$ kg for Earth. A larger center-to-center distance weakens gravity.

3. The same gravitational pull also fixes the speed of a circular orbit. Set the two expressions for inward force equal: $mv^2/r = GMm/r^2$. Cancel the spacecraft's mass and simplify to $v^2 = GM/r$, or $v = \sqrt{(GM/r)}$. This model predicts that, around the same planet, a more distant circular orbit has a lower speed. The mass of the spacecraft changes the gravitational force, but it does not change the circular-orbit speed at a given radius. If the distance between centers is scaled up, compare forces using the square of that scale factor. These equations assume that gravity is the only force acting.

2 ANSWER WITH EVIDENCE

Use the passage

1. A 40 kg cart moves at 5 m/s on a level circular track of radius 10 m. Calculate its centripetal acceleration and inward net force.

2. Sketch a free-body diagram for a 1,000 kg spacecraft 2.0×10^7 m from Earth's center. Label the force arrow with its direction and calculate its magnitude.

3. Two probes orbit the same planet. Probe B has twice probe A's mass and is three times as far from the planet's center. What is F_B/F_A for their gravitational forces?

4. What circular-orbit speed does the model predict for a spacecraft 1.0×10^7 m from Earth's center? Show the equation you use.

3 YOUR TURN

Your own words

In the two panels, model equal-mass spacecraft in circular orbits around the same planet, one at radius r and one at radius 4r. Draw and label each spacecraft's free-body diagram and tangential velocity, then write equations comparing their gravitational forces and orbital speeds and state the direction of their accelerations.

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PART 1 · READ AND MARK

Underline the equations that connect speed, radius, acceleration, and force. Circle each statement that tells you which way to draw an arrow in a model of circular motion. Accept any marking that follows this job and points at the right text.

PART 2 · ANSWERS

#	QUESTION	ANSWER
1	A 40 kg cart moves at 5 m/s on a level circular track of radius 10 m. Calculate its centripetal acceleration and inward net force.	**$a_c = v^2/r = 2.5\ m/s^2$; $F_{net} = ma_c = 100\ N$ inward.**
2	Sketch a free-body diagram for a 1,000 kg spacecraft 2.0×10^7 m from Earth's center. Label the force arrow with its direction and calculate its magnitude.	**One gravitational-force arrow points toward Earth's center. $F_g = GMm/r^2 \approx 1.0 \times 10^3$ N.**
3	Two probes orbit the same planet. Probe B has twice probe A's mass and is three times as far from the planet's center. What is F_B/F_A for their gravitational forces?	**$F_B/F_A = 2/3^2 = 2/9$.**
4	What circular-orbit speed does the model predict for a spacecraft 1.0×10^7 m from Earth's center? Show the equation you use.	**$v = \sqrt{(GM/r)} = \sqrt{[(6.67 \times 10^{-11})(6.0 \times 10^{24})/(1.0 \times 10^7)]} \approx 6.3 \times 10^3$ m/s.**

PART 3 · YOUR TURN (SAMPLE ANSWER)

Radius r panel: The planet's center is to the right; a single F_g arrow points right and a v arrow points up. Radius 4r panel: The planet's center is to the right; a single F_g arrow points right and a v arrow points up. $F_r/F_{4r} = 16$, and $v_r/v_{4r} = 2$. Both accelerations point toward the planet's center.

Accept any orientation with each gravitational-force and acceleration direction pointing toward the planet's center and each velocity tangent to its orbit. The force ratio is 16 because force varies as $1/r^2$; the speed ratio is 2 because orbital speed varies as $1/\sqrt{r}$.