

Remainder Detectives

Evaluate polynomials to find remainders and factors.

HIGH SCHOOL • HSA-APR.B.2

CCSS.Math.Content.HSA-APR.B.2 "Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$."

NAME _____

DATE _____

THE REMAINDER SHORTCUT

When dividing $p(x)$ by $x - a$, plug a into $p(x)$. The value $p(a)$ is the remainder. If $p(a) = 0$, then $x - a$ is a factor of $p(x)$.

WORKED EXAMPLE

Find the remainder when $p(x) = x^4 + 13x - 11$ is divided by $x + 2$.

1. Write $x + 2$ as $x - (-2)$, so $a = -2$.
2. Evaluate $p(-2) = (-2)^4 + 13(-2) - 11$.
3. Simplify: $16 - 26 - 11 = -21$.

Answer: The remainder is -21.

PRACTICE 6 problems

- 1** Find the remainder when $p(x) = x^2 - 6x + 10$ is divided by $x - 4$.

ANSWER _____

- 2** Find the remainder when $p(x) = 3x^3 + x^2 - 12$ is divided by $x + 1$.

ANSWER _____

- 3** Is $x - 6$ a factor of $p(x) = x^3 - 2x^2 - 24x$? Give the remainder and conclusion.

ANSWER _____

- 4** Find k so that $x + 4$ is a factor of $p(x) = x^2 + kx - 20$.

ANSWER _____

- 5** Is $x + 6$ a factor of $p(x) = x^3 - 36x + 8$? Give the remainder and conclusion.

ANSWER _____

- 6** Find the remainder when $p(x) = 4x^3 + 5x^2 - 8x + 9$ is divided by $x - 3$.

ANSWER _____

APPLY IT Show your work

- 1** A coding club models a rhythm game's bonus-score formula with $B(x) = x^3 - 9x^2 + 14x + 24$. The club wants to know whether $x - 4$ is a factor before simplifying the formula. Find the remainder and state the conclusion.

WORK SPACE

ANSWER _____

- 2** The student council's concert-planning spreadsheet uses $R(x) = 2x^3 - 7x^2 - 3x + 18$ as a test revenue expression. The team checks whether $x - 4$ is a factor. Find the remainder and state the conclusion.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

The polynomial $p(x) = x^3 + bx^2 - 15x + 45$ has $x - 3$ as a factor. Find b . Then decide whether $x + 5$ is a factor of the resulting polynomial. Show the values you evaluate.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Find the remainder when $p(x) = x^2 - 6x + 10$ is divided by $x - 4$.	2
2	Find the remainder when $p(x) = 3x^3 + x^2 - 12$ is divided by $x + 1$.	-14
3	Is $x - 6$ a factor of $p(x) = x^3 - 2x^2 - 24x$? Give the remainder and conclusion.	The remainder is 0, so $x - 6$ is a factor.
4	Find k so that $x + 4$ is a factor of $p(x) = x^2 + kx - 20$.	$k = -1$
5	Is $x + 6$ a factor of $p(x) = x^3 - 36x + 8$? Give the remainder and conclusion.	The remainder is 8, so $x + 6$ is not a factor.
6	Find the remainder when $p(x) = 4x^3 + 5x^2 - 8x + 9$ is divided by $x - 3$.	138

PART 2 • APPLY IT

- The remainder is 0, so $x - 4$ is a factor of $B(x)$.** Evaluate $B(4) = 4^3 - 9(4^2) + 14(4) + 24 = 64 - 144 + 56 + 24 = 0$. A remainder of 0 means $x - 4$ is a factor.
- The remainder is 22, so $x - 4$ is not a factor of $R(x)$.** Evaluate $R(4) = 2(4^3) - 7(4^2) - 3(4) + 18 = 128 - 112 - 12 + 18 = 22$. Since the remainder is not 0, $x - 4$ is not a factor.

CHALLENGE

$b = -3$. The resulting polynomial is $p(x) = x^3 - 3x^2 - 15x + 45$. Since $p(-5) = -80$, $x + 5$ is not a factor.

Accept correct substitution using $a = -c$ when the divisor is $x + c$. For factor questions, students should state both the remainder and the matching factor or nonfactor conclusion.