Zero In

Find intercepts, then use them to shape a graph.

HIGH SCHOOL · HSA-APR.B.3

CCSS.Math.Content.HSA-APR.B.3 "Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial."

NAME ______________________ DATE ____________

FACTORS SHOW ZEROS

A zero is an x-value that makes a polynomial equal 0, so it gives an x-intercept on the graph. In factored form, set each factor equal to 0. A factor with even multiplicity touches the x-axis, while a factor with odd multiplicity crosses it; the leading term tells how the ends of the graph behave.

WORKED EXAMPLE

Find the zeros of $F(x) = (x - 7)(x + 9)$, then describe a rough graph.

1. Set each factor equal to 0.
2. $x - 7 = 0$ gives $x = 7$.
3. $x + 9 = 0$ gives $x = -9$.
4. The leading coefficient is positive, so the parabola opens up and crosses at both zeros.

Answer: Zeros: $x = -9$ and $x = 7$. The graph is an upward-opening parabola with x-intercepts (-9, 0) and (7, 0).

PRACTICE

6 problems

1 Find the zeros of $f(x) = (x - 2)(x + 5)$. Then describe a rough graph.

ANSWER ______________________

2 Find the zeros of $g(x) = -(x + 1)(x - 4)$. Then describe a rough graph.

ANSWER ______________________

3 Find the zeros of $h(x) = (x - 3)^2(x + 2)$. Then describe a rough graph.

ANSWER ______________________

4 Find the zeros of $p(x) = (2x - 1)(x + 3)$. Then describe a rough graph.

ANSWER ______________________

5 Find the zeros of $q(x) = -(x - 6)^2(x + 1)$. Then describe a rough graph.

ANSWER ______________________

6 Find the zeros of $r(x) = (x + 4)(x - 1)(x - 2)$. Then describe a rough graph.

ANSWER ______________________

APPLY IT
Show your work

1 A basketball's height, in feet, is modeled by $h(t) = -2(t - 1)(t - 4)$, where t is time in seconds. Find when the ball is at floor level. Then describe a rough graph for $1 <= t <= 4$.

WORK SPACE

ANSWER ______________________

2 A student concert committee models its net profit, in hundreds of dollars, by $P(d) = (d + 3)(d - 2)(d - 5)$, where d is the change in ticket price, in dollars, from the planned price. Find all break-even price changes. Then describe a rough graph of $P(d)$.

WORK SPACE

ANSWER ______________________

CHALLENGE
One step up

CHALLENGE

Without multiplying, analyze $k(x) = -(x + 2)^2(x - 3)(x - 5)^3$. Identify each zero and its multiplicity. Then explain where the graph crosses or touches the x-axis and describe both ends of the graph.

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ANSWER KEY # Zero In

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the zeros of f(x) = (x - 2)(x + 5). Then describe a rough graph.	**Zeros: x = -5 and x = 2. The graph is an upward-opening parabola that crosses the x-axis at both intercepts.**
2	Find the zeros of g(x) = -(x + 1)(x - 4). Then describe a rough graph.	**Zeros: x = -1 and x = 4. The graph is a downward-opening parabola that crosses the x-axis at both intercepts.**
3	Find the zeros of h(x) = (x - 3)^2(x + 2). Then describe a rough graph.	**Zeros: x = -2 and x = 3, with x = 3 having multiplicity 2. The graph crosses at x = -2, touches and turns at x = 3, falls to the left, and rises to the right.**
4	Find the zeros of p(x) = (2x - 1)(x + 3). Then describe a rough graph.	**Zeros: x = -3 and x = 1/2. The graph is an upward-opening parabola that crosses the x-axis at both intercepts.**
5	Find the zeros of q(x) = -(x - 6)^2(x + 1). Then describe a rough graph.	**Zeros: x = -1 and x = 6, with x = 6 having multiplicity 2. The graph crosses at x = -1, touches and turns at x = 6, rises to the left, and falls to the right.**
6	Find the zeros of r(x) = (x + 4)(x - 1)(x - 2). Then describe a rough graph.	**Zeros: x = -4, x = 1, and x = 2. The graph crosses the x-axis at all three zeros, falls to the left, and rises to the right.**

PART 2 · APPLY IT

1. **The ball is at floor level at t = 1 second and t = 4 seconds. The graph is a downward-opening parabola that crosses the t-axis at 1 and 4 and has a maximum height of 9/2 feet at t = 5/2 seconds.** Set each factor equal to 0 to get t = 1 and t = 4. The negative leading coefficient means the parabola opens down; the vertex is halfway between the zeros.
2. **The break-even price changes are d = -3, d = 2, and d = 5. A change of d = -3 means lowering the ticket price by $3. The cubic graph crosses the d-axis at all three values, falls to the left, and rises to the right.** Set each factor equal to 0 to find the three break-even values. Each factor occurs once, so the graph crosses at every zero, and the positive cubic leading term gives left-down, right-up end behavior.

CHALLENGE

The zeros are x = -2 with multiplicity 2, x = 3 with multiplicity 1, and x = 5 with multiplicity 3. The graph touches and turns at x = -2 because the multiplicity is even. It crosses at x = 3 and x = 5 because those multiplicities are odd, and it flattens as it crosses at x = 5 because the multiplicity is 3. The polynomial has degree 6 with a negative leading coefficient, so both ends fall.

Accept zeros in any order and as x-values or x-intercepts with y = 0. For graph descriptions, accept equivalent wording when it correctly states intercepts, crossing or touching behavior, and end behavior.