LESSON MAKER PRO

# Identity Triple Lab

*Expand identities and build whole-number right-triangle relationships.*

HIGH SCHOOL · HSA-APR.C.4

**CCSS.Math.Content.HSA-APR.C.4** “Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity (x 2 + y 2 ) 2 = (x 2 - y 2 ) 2 + (2xy) 2 can be used to generate Pythagorean triples.”

NAME ____________________ DATE ____________

### A USEFUL SQUARE IDENTITY

The identity $(a^2 - b^2)^2 + (2ab)^2 = (a^2 + b^2)^2$ is true for all values of a and b. Prove identities by expanding each side and combining like terms. When a and b are positive integers with a greater than b, this identity produces three whole numbers with a special square relationship.

### WORKED EXAMPLE

Use a = 8 and b = 3 to generate and verify a whole-number triple from the identity.

1. $a^2 - b^2 = 64 - 9 = 55$, and $2ab = 48$.
2. $a^2 + b^2 = 64 + 9 = 73$.
3. $55^2 + 48^2 = 3025 + 2304 = 5329$.
4. $73^2 = 5329$, so $55^2 + 48^2 = 73^2$.

**Answer: 55, 48, 73**

## PRACTICE 6 problems

**1** Verify by expanding: $(x + 4)^2 - (x - 4)^2 = 16x$.

ANSWER ____________________

**2** Factor $z^4 - 49$ as a product of two polynomials by using a difference of squares identity.

ANSWER ____________________

**3** Prove that $(m^2 + n^2)^2 - (m^2 - n^2)^2 = 4m^2n^2$ by expanding both squared expressions.

ANSWER ____________________

**4** Use a = 6 and b = 1 in the identity pattern. List the three whole numbers that satisfy the square relationship.

ANSWER ____________________

**5** Use a = 12 and b = 5 in the triple identity to verify that $119^2 + 120^2 = 169^2$ without first multiplying 119 by 119.

ANSWER ____________________

**6** Use a = 7 and b = 4 in the identity pattern. List the three whole numbers that satisfy the square relationship.

ANSWER ____________________

## APPLY IT
Show your work

**1** A robotics club programs a rover to travel along two perpendicular paths and then return directly to its starting point. The club chooses $a = 10$ and $b = 1$ in the identity pattern. What whole-number lengths does the pattern give for the two paths and the direct return? Verify the square relationship.

WORK SPACE

**ANSWER** ____________________

**2** An esports team is designing a rectangular practice map with whole-number side lengths and a whole-number diagonal. They choose $a = 13$ and $b = 6$ in the identity pattern. What are the two side lengths and the diagonal? Verify the square relationship.

WORK SPACE

**ANSWER** ____________________

## CHALLENGE
One step up

**CHALLENGE**

For positive integers a and b with a greater than b, use expansion to prove that a^2 - b^2, 2ab, and a^2 + b^2 always satisfy a Pythagorean relationship. Explain why these expressions form a whole-number triple.

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**ANSWER KEY** # Identity Triple Lab

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## PART 1 · PRACTICE

| # | PROBLEM | ANSWER |
|---|---|---|
| 1 | Verify by expanding: $(x + 4)^2 - (x - 4)^2 = 16x$. | **$(x^2 + 8x + 16) - (x^2 - 8x + 16) = 16x$, so the identity is true.** |
| 2 | Factor $z^4 - 49$ as a product of two polynomials by using a difference of squares identity. | **$(z^2 - 7)(z^2 + 7)$** |
| 3 | Prove that $(m^2 + n^2)^2 - (m^2 - n^2)^2 = 4m^2n^2$ by expanding both squared expressions. | **$(m^4 + 2m^2n^2 + n^4) - (m^4 - 2m^2n^2 + n^4) = 4m^2n^2$** |
| 4 | Use a = 6 and b = 1 in the identity pattern. List the three whole numbers that satisfy the square relationship. | **35, 12, 37** |
| 5 | Use a = 12 and b = 5 in the triple identity to verify that $119^2 + 120^2 = 169^2$ without first multiplying 119 by 119. | **$12^2 - 5^2 = 119$, $2(12)(5) = 120$, and $12^2 + 5^2 = 169$. Therefore, $119^2 + 120^2 = 169^2$.** |
| 6 | Use a = 7 and b = 4 in the identity pattern. List the three whole numbers that satisfy the square relationship. | **33, 56, 65** |

## PART 2 · APPLY IT

1. **The two path lengths are 99 and 20, and the direct return is 101. $99^2 + 20^2 = 9801 + 400 = 10201 = 101^2$.** Compute $a^2 - b^2$, $2ab$, and $a^2 + b^2$. Substitute 10 and 1 to get 99, 20, and 101, then compare the squares.
2. **The side lengths are 133 and 156, and the diagonal is 205. $133^2 + 156^2 = 17689 + 24336 = 42025 = 205^2$.** Use $a^2 - b^2$ and $2ab$ for the side lengths, and use $a^2 + b^2$ for the diagonal. The identity guarantees that the sum of the first two squares equals the third square.

## CHALLENGE

**$(a^2 - b^2)^2 + (2ab)^2 = a^4 - 2a^2b^2 + b^4 + 4a^2b^2 = a^4 + 2a^2b^2 + b^4 = (a^2 + b^2)^2$. Since a and b are positive integers and a > b, all three expressions are positive integers, so they form a Pythagorean triple.**

Accept equivalent expanded forms and factored forms with factors in either order. For generated triples, accept the two shorter values in either order when the student verifies the square relationship.