

Binomial Power Play

Use Pascal's Triangle to expand powers of binomials.

HIGH SCHOOL • HSA-APR.C.5

CCSS.Math.Content.HSA-APR.C.5 "(+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n , where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. 1"

NAME _____

DATE _____

BUILD EACH TERM

For $(x + y)^n$, use the coefficients in row n of Pascal's Triangle, with the top 1 called row 0. Write the power of x down from n to 0 and the power of y up from 0 to n . If a binomial includes a negative term, the signs alternate, and any coefficient attached to a variable is raised with that variable.

WORKED EXAMPLE

Expand $(u + v)^8$.

1. Pascal's Triangle row 8 is 1, 8, 28, 56, 70, 56, 28, 8, 1.
2. Write powers of u from u^8 to u^0 and powers of v from v^0 to v^8 .
3. Match each coefficient to its pair of variable powers.

Answer: $u^8 + 8u^7v + 28u^6v^2 + 56u^5v^3 + 70u^4v^4 + 56u^3v^5 + 28u^2v^6 + 8uv^7 + v^8$

PRACTICE 6 problems

1 Expand $(a + b)^2$.

ANSWER _____

2 Expand $(m - n)^3$.

ANSWER _____

3 Expand $(c + d)^4$.

ANSWER _____

4 What is the coefficient of p^3q^2 in $(p + q)^5$?

ANSWER _____

5 Expand $(3x + y)^3$.

ANSWER _____

6 Expand $(r - 3s)^4$.

ANSWER _____

APPLY IT Show your work

- 1** A game developer's score model is $S = (g + 4)^4$, where g represents a bonus multiplier from a player's choices. Write S in expanded form.

WORK SPACE

ANSWER _____

- 2** A school photography club tests an editing filter with output score $F = (p - 5)^3$, where p is a brightness setting. Write F in expanded form.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Without fully expanding $(2x - 3y)^7$, find the coefficient of x^4y^3 . Explain your reasoning.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Expand $(a + b)^2$.	$a^2 + 2ab + b^2$
2	Expand $(m - n)^3$.	$m^3 - 3m^2n + 3mn^2 - n^3$
3	Expand $(c + d)^4$.	$c^4 + 4c^3d + 6c^2d^2 + 4cd^3 + d^4$
4	What is the coefficient of p^3q^2 in $(p + q)^5$?	10
5	Expand $(3x + y)^3$.	$27x^3 + 27x^2y + 9xy^2 + y^3$
6	Expand $(r - 3s)^4$.	$r^4 - 12r^3s + 54r^2s^2 - 108rs^3 + 81s^4$

PART 2 · APPLY IT

1. **$S = g^4 + 16g^3 + 96g^2 + 256g + 256$** Use the row 4 coefficients 1, 4, 6, 4, 1. Treat 4 as the second binomial term in each product.
2. **$F = p^3 - 15p^2 + 75p - 125$** Use the row 3 coefficients 1, 3, 3, 1. Because the second term is negative, the signs alternate.

CHALLENGE

There are 35 ways to choose the three y factors. The coefficient is $35(2^4)(-3)^3 = -15120$.

Accept equivalent expanded forms with terms written in a different order. For expansions with subtraction, accept correct alternating signs and omitted factors of 1.