LESSON MAKER PRO

Square It Out

Transform quadratic equations and solve them.

HIGH SCHOOL · HSA-REI.B.4a

CCSS.Math.Content.HSA-REI.B.4a "Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x - p) 2 = q that has the same solutions. Derive the quadratic formula from this form."

NAME _____ DATE _____

COMPLETE THE SQUARE

First make the coefficient of x^2 equal to 1, then move the constant term to the other side. Add the square of half the x-coefficient to both sides so the left side becomes a squared binomial. Using this process with ax^2 + bx + c = 0 leads to the quadratic formula.

WORKED EXAMPLE

Solve x^2 + 14x - 13 = 0 by completing the square.

1. x^2 + 14x = 13
2. Add 49 to both sides: x^2 + 14x + 49 = 62
3. (x + 7)^2 = 62, so x = -7 +/- sqrt(62)

Answer: x = -7 +/- sqrt(62)

PRACTICE 6 problems

1 Solve x^2 + 8x + 12 = 0 by completing the square.

ANSWER _____

2 Solve x^2 - 6x - 16 = 0 by completing the square.

ANSWER _____

3 Solve 2x^2 + 12x - 32 = 0 by completing the square.

ANSWER _____

4 Solve 3x^2 - 18x + 15 = 0 by completing the square.

ANSWER _____

5 Solve x^2 + 2x - 3 = 0 by completing the square.

ANSWER _____

6 Solve 4x^2 + 8x - 8 = 0 by completing the square.

ANSWER _____

APPLY IT Show your work

1 A student launches a water rocket from a 20-foot platform. Its height h, in feet, after t seconds is h = -t^2 + 8t + 20. Use completing the square to find when the rocket hits the ground.

WORK SPACE

ANSWER _____

2 The drama club models its profit P, in dollars, from a ticket price p, in dollars, with P = -2p^2 + 24p - 64. At which ticket prices does the club break even?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Starting with ax^2 + bx + c = 0, where a != 0, use completing the square to derive the quadratic formula. Write the equations that lead to the formula.

_____

_____

ANSWER KEY

Square It Out

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Solve x^2 + 8x + 12 = 0 by completing the square.	**(x + 4)^2 = 4; x = -6, -2**
2	Solve x^2 - 6x - 16 = 0 by completing the square.	**(x - 3)^2 = 25; x = -2, 8**
3	Solve 2x^2 + 12x - 32 = 0 by completing the square.	**(x + 3)^2 = 25; x = -8, 2**
4	Solve 3x^2 - 18x + 15 = 0 by completing the square.	**(x - 3)^2 = 4; x = 1, 5**
5	Solve x^2 + 2x - 3 = 0 by completing the square.	**(x + 1)^2 = 4; x = -3, 1**
6	Solve 4x^2 + 8x - 8 = 0 by completing the square.	**(x + 1)^2 = 3; x = -1 +/- sqrt(3)**

PART 2 · APPLY IT

1. **The rocket hits the ground at t = 10 seconds.** Set h equal to 0 and rewrite the equation as t^2 - 8t - 20 = 0. Completing the square gives (t - 4)^2 = 36, so t = 10 or t = -2; only 10 seconds is meaningful.
2. **The club breaks even when p = 4 dollars or p = 8 dollars.** Set P equal to 0, then divide by -2 to get p^2 - 12p + 32 = 0. Completing the square gives (p - 6)^2 = 4, so p = 4 or p = 8.

CHALLENGE

x^2 + (b/a)x = -c/a; (x + b/(2a))^2 = (b^2 - 4ac)/(4a^2); x = (-b +/- sqrt(b^2 - 4ac))/(2a)

Accept equivalent completed-square equations and either order of the solutions. For equations with a leading coefficient other than 1, students should divide by that coefficient before completing the square.