Same Solution Swap

Replace an equation without changing the answers.

HIGH SCHOOL · HSA-REI.C.5

CCSS.Math.Content.HSA-REI.C.5 "Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions."

NAME ______________________ DATE __________

ONE SAFE MOVE

You may replace one equation by itself plus k times the other equation. Every original solution makes the new equation true, and you can undo the move by subtracting k times the unchanged equation.

WORKED EXAMPLE

Replace E1 with E1 + 3E2. E1: $9x + 5y = 41$ E2: $2x - y = 8$

1. Write E1 + 3E2: $(9x + 5y) + 3(2x - y) = 41 + 3(8)$.
2. Combine like terms: $15x + 2y = 65$.
3. Keep E2 unchanged.

Answer: New system: $15x + 2y = 65$ and $2x - y = 8$

PRACTICE

5 problems

1 Replace E1 with E1 - 2E2. E1: $x + 4y = 10$ E2: $6x - y = 12$

ANSWER ______________________

2 Replace E2 with E2 + 4E1. E1: $4x + y = 13$ E2: $x - 2y = 1$

ANSWER ______________________

3 Replace E2 with E2 + 2E1. E1: $5x - y = 16$ E2: $2x + 4y = 18$

ANSWER ______________________

4 Replace E1 with E1 + 5E2. Then solve. E1: $2x + 5y = 19$ E2: $3x - y = 3$

ANSWER ______________________

5 Replace E2 with E2 - 3E1. E1: $x + 2y = 9$ E2: $4x + 5y = 22$

ANSWER ______________________

APPLY IT
Show your work

1 A gaming club sells 18 tickets. Student tickets cost \$3 and guest tickets cost \$5, for \$74 total. Let s be student tickets and g be guest tickets. Use the total equation to replace the money equation, then find s and g.

WORK SPACE

ANSWER ______________________

2 A school store sells 12 hoodies. Standard hoodies cost \$4 and premium hoodies cost \$9, for \$68 total. Let s be standard hoodies and p be premium hoodies. Use the total equation to replace the money equation, then find s and p.

WORK SPACE

ANSWER ______________________

CHALLENGE
One step up

CHALLENGE

Let E1 be A(x,y) = b and E2 be C(x,y) = d. E1 is replaced by A(x,y) + kC(x,y) = b + kd. In 2 sentences, explain why the systems have the same solutions.

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Same Solution Swap

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Replace E1 with E1 - 2E2. E1: x + 4y = 10 E2: 6x - y = 12	**New E1: -11x + 6y = -14**
2	Replace E2 with E2 + 4E1. E1: 4x + y = 13 E2: x - 2y = 1	**New E2: 17x + 2y = 53**
3	Replace E2 with E2 + 2E1. E1: 5x - y = 16 E2: 2x + 4y = 18	**New E2: 12x + 2y = 50**
4	Replace E1 with E1 + 5E2. Then solve. E1: 2x + 5y = 19 E2: 3x - y = 3	**New E1: 17x = 34; x = 2, y = 3**
5	Replace E2 with E2 - 3E1. E1: x + 2y = 9 E2: 4x + 5y = 22	**New E2: x - y = -5**

PART 2 · APPLY IT

1. **s = 8, g = 10** Use s + g = 18 and 3s + 5g = 74. Replace the money equation with (3s + 5g) - 3(s + g) = 74 - 54, so 2g = 20.
2. **s = 8, p = 4** Use s + p = 12 and 4s + 9p = 68. Replace the money equation with (4s + 9p) - 4(s + p) = 68 - 48, so 5p = 20.

CHALLENGE

If a pair satisfies A(x,y) = b and C(x,y) = d, then it satisfies A(x,y) + kC(x,y) = b + kd. If it satisfies C(x,y) = d and A(x,y) + kC(x,y) = b + kd, subtracting k times C(x,y) = d gives A(x,y) = b.

Accept equivalent replacement equations, including equations multiplied by a nonzero constant. For the challenge, accept a two-direction argument showing that the replacement can be undone.