

Matrix Mission

Use inverses to unlock linear systems.

HIGH SCHOOL • HSA-REI.C.9

CCSS.Math.Content.HSA-REI.C.9 "(+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 $\times$ 3 or greater)."

NAME _____

DATE _____

INVERT, THEN SOLVE

For a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the inverse is $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, as long as $ad-bc$ is not 0. To solve $AX = B$, multiply both sides by A^{-1} , so $X = A^{-1}B$.

WORKED EXAMPLE

Find the inverse of $A = \begin{bmatrix} 13 & 8 \\ 21 & 13 \end{bmatrix}$.

1. $\det(A) = 13(13) - 8(21) = 1$.

2. $A^{-1} = \frac{1}{1} \begin{bmatrix} 13 & -8 \\ -21 & 13 \end{bmatrix}$.

3. So $A^{-1} = \begin{bmatrix} 13 & -8 \\ -21 & 13 \end{bmatrix}$.

Answer: $\begin{bmatrix} 13 & -8 \\ -21 & 13 \end{bmatrix}$

PRACTICE 6 problems

1 Find the inverse of $A = \begin{bmatrix} 4 & 1 \\ 15 & 4 \end{bmatrix}$.

ANSWER _____

2 Does $A = \begin{bmatrix} 6 & 9 \\ 4 & 6 \end{bmatrix}$ have an inverse?

ANSWER _____

3 Use an inverse matrix to solve the system: $x + 4y = 9$ and $6x + 25y = 56$.

ANSWER _____

4 Use an inverse matrix to solve the system: $4x + 6y = 36$ and $x + 2y = 11$.

ANSWER _____

5 Find the inverse of $A = \begin{bmatrix} 9 & 1 \\ 17 & 2 \end{bmatrix}$.

ANSWER _____

6 Can the inverse matrix method be used to solve $4x + 6y = 18$ and $2x + 3y = 9$? Then solve the system.

ANSWER _____

APPLY IT Show your work

- 1** At a school esports event, a student buys team patches for \$6 each and buttons for \$2 each. The student buys 4 items for \$12. Use a matrix inverse to find how many patches and buttons the student buys.

WORK SPACE

ANSWER _____

- 2** A robotics team completes design challenges worth 4 points each and outreach events worth 1 point each. The team completes 9 tasks and earns 24 points. Use a matrix inverse to find the number of each task.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Let $A = \begin{bmatrix} k & 4 \\ 6 & 8 \end{bmatrix}$. For what value of k does A have no inverse? At that value, describe the solutions to the system $3x + 4y = 11$ and $6x + 8y = 22$.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Find the inverse of $A = \begin{bmatrix} 4 & 1 \\ 15 & 4 \end{bmatrix}$.	$\begin{bmatrix} 4 & -1 \\ -15 & 4 \end{bmatrix}$
2	Does $A = \begin{bmatrix} 6 & 9 \\ 4 & 6 \end{bmatrix}$ have an inverse?	No. $\det(A) = 0$, so the inverse does not exist.
3	Use an inverse matrix to solve the system: $x + 4y = 9$ and $6x + 25y = 56$.	$x = 1, y = 2$
4	Use an inverse matrix to solve the system: $4x + 6y = 36$ and $x + 2y = 11$.	$x = 3, y = 4$
5	Find the inverse of $A = \begin{bmatrix} 9 & 1 \\ 17 & 2 \end{bmatrix}$.	$\begin{bmatrix} 2 & -1 \\ -17 & 9 \end{bmatrix}$
6	Can the inverse matrix method be used to solve $4x + 6y = 18$ and $2x + 3y = 9$? Then solve the system.	No, because the coefficient matrix has determinant 0. There are infinitely many solutions: $(x,y) = (9/2 - 3t/2, t)$, for any real number t .

PART 2 • APPLY IT

- 1 patch and 3 buttons** Let p be patches and b be buttons. Solve $\begin{bmatrix} 1 & 1 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} p \\ b \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \end{bmatrix}$ to get $p = 1$ and $b = 3$.
- 5 design challenges and 4 outreach events** Let d be design challenges and o be outreach events. Solve $\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} d \\ o \end{bmatrix} = \begin{bmatrix} 9 \\ 24 \end{bmatrix}$ to get $d = 5$ and $o = 4$.

CHALLENGE

$k = 3$. At $k = 3$, the system has infinitely many solutions because the second equation is twice the first equation.

Accept equivalent inverse notation and equivalent parameter forms for infinitely many solutions. For inverse questions, students may show a nonzero determinant and use the 2×2 inverse formula in any correct order.