

Vertex Value Vault

Complete the square to uncover each highest or lowest value.

HIGH SCHOOL • HSA-SSE.B.3b

CCSS.Math.Content.HSA-SSE.B.3b "Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines."

NAME _____

DATE _____

REVEAL THE EXTREME

Rewrite a quadratic so it has a squared term plus or minus a constant. Because a square is never negative, a positive squared term reveals a minimum, and a negative squared term reveals a maximum.

WORKED EXAMPLE

Complete the square and find the minimum value of $f(x) = x^2 + 10x + 1$.

1. Group the quadratic and linear terms:
 $f(x) = (x^2 + 10x) + 1$.
2. Complete the square: $x^2 + 10x = (x + 5)^2 - 25$.
3. Substitute: $f(x) = (x + 5)^2 - 24$.
4. Since $(x + 5)^2$ is at least 0, the minimum is -24 when $x = -5$.

Answer: $f(x) = (x + 5)^2 - 24$; minimum value: -24 at $x = -5$

PRACTICE 6 problems

- 1** Complete the square and find the minimum value of $f(x) = x^2 + 6x + 11$.

ANSWER _____

- 2** Complete the square and find the minimum value of $g(x) = x^2 - 14x + 20$.

ANSWER _____

- 3** Complete the square and find the maximum value of $h(x) = -x^2 + 4x + 9$.

ANSWER _____

- 4** Complete the square and find the maximum value of $p(x) = -x^2 - 12x - 8$.

ANSWER _____

- 5** Complete the square and find the minimum value of $q(x) = 2x^2 + 14x + 6$.

ANSWER _____

- 6** Complete the square and find the maximum value of $r(x) = -3x^2 + 18x - 4$.

ANSWER _____

APPLY IT Show your work

- 1** A skateboarder's height, $h(t)$, in feet is modeled by $h(t) = -7t^2 + 28t + 6$, where t is time in seconds after takeoff. Complete the square, then find the skateboarder's maximum height and the time when it occurs.

WORK SPACE

ANSWER _____

- 2** A school music club models its profit, $P(x)$, in tens of dollars by $P(x) = -2x^2 + 16x + 3$. Here, x is the number of 1-dollar ticket price increases from the current ticket price. Complete the square, then find the price increase that gives the greatest profit and the greatest profit.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Complete the square for $m(x) = 3x^2 - 9x + 2$. Then state the minimum value and explain why it is a minimum.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Complete the square and find the minimum value of $f(x) = x^2 + 6x + 11$.	$f(x) = (x + 3)^2 + 2$; minimum value: 2 at $x = -3$
2	Complete the square and find the minimum value of $g(x) = x^2 - 14x + 20$.	$g(x) = (x - 7)^2 - 29$; minimum value: -29 at $x = 7$
3	Complete the square and find the maximum value of $h(x) = -x^2 + 4x + 9$.	$h(x) = -(x - 2)^2 + 13$; maximum value: 13 at $x = 2$
4	Complete the square and find the maximum value of $p(x) = -x^2 - 12x - 8$.	$p(x) = -(x + 6)^2 + 28$; maximum value: 28 at $x = -6$
5	Complete the square and find the minimum value of $q(x) = 2x^2 + 14x + 6$.	$q(x) = 2(x + 7/2)^2 - 37/2$; minimum value: -37/2 at $x = -7/2$
6	Complete the square and find the maximum value of $r(x) = -3x^2 + 18x - 4$.	$r(x) = -3(x - 3)^2 + 23$; maximum value: 23 at $x = 3$

PART 2 • APPLY IT

- $h(t) = -7(t - 2)^2 + 34$; maximum height: 34 feet at 2 seconds** Factor -7 from the quadratic and linear terms, then complete the square. The negative squared term is greatest when $(t - 2)^2 = 0$.
- $P(x) = -2(x - 4)^2 + 35$; greatest profit: \$350 with a \$4 price increase** Rewrite the function as $-2(x - 4)^2 + 35$. The maximum model value is 35 tens of dollars when $x = 4$.

CHALLENGE

$m(x) = 3(x - 3/2)^2 - 19/4$; minimum value: -19/4 at $x = 3/2$. It is a minimum because $3(x - 3/2)^2$ is always greater than or equal to 0.

Accept equivalent completed-square forms, including reordered terms. Students should identify both the extreme value and the x -value where it occurs.