

Sequence Switchboard

Move between recursive rules, explicit formulas, and real situations.

HIGH SCHOOL • HSF-BF.A.2

CCSS.Math.Content.HSF-BF.A.2 "Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. *"

NAME _____

DATE _____

TWO WAYS TO DESCRIBE A SEQUENCE

An arithmetic sequence changes by adding or subtracting the same difference each time. A geometric sequence changes by multiplying by the same ratio each time. A recursive rule gives a starting term and a rule using the previous term, while an explicit formula gives any term directly from its position.

WORKED EXAMPLE

An arithmetic sequence has $a_1 = 11$ and common difference -8 . Write a recursive rule and an explicit formula.

1. Start with $a_1 = 11$.
2. Subtract 8 from each previous term: $a_n = a_{(n-1)} - 8$ for $n \geq 2$.
3. Use $a_n = 11 + (n-1)(-8)$.
4. Simplify: $a_n = 19 - 8n$.

Answer: Recursive: $a_1 = 11$, $a_n = a_{(n-1)} - 8$ for $n \geq 2$. Explicit: $a_n = 11 + (n-1)(-8)$, or $a_n = 19 - 8n$.

PRACTICE 6 problems

- 1** An arithmetic sequence has $a_1 = 5$ and common difference 6. Write a recursive rule and an explicit formula.

ANSWER _____

- 2** A geometric sequence has $b_1 = 2$ and common ratio 4. Write a recursive rule and an explicit formula.

ANSWER _____

- 3** The explicit formula is $c_n = 9 - 2(n-1)$. Write a recursive rule. Then list the first four terms.

ANSWER _____

- 4** The explicit formula is $d_n = 3(5)^{(n-1)}$. Write a recursive rule and find d_4 .

ANSWER _____

- 5** The sequence begins 64, 32, 16, ... Write a recursive rule and an explicit formula.

ANSWER _____

- 6** A sequence is defined by $e_1 = -4$ and $e_n = e_{(n-1)} + 7$ for $n \geq 2$. Write an explicit formula and find e_6 .

ANSWER _____

APPLY IT Show your work

- 1** A student council scholarship fund has \$120 at the end of week 1. The council adds \$35 each week. Let F_n represent the total amount in the fund at the end of week n . Write a recursive rule and an explicit formula. How much will be in the fund at the end of week 6?

WORK SPACE

ANSWER _____

- 2** A school club's video receives 7 shares on day 1. The number of shares each day is twice the number from the day before. Let S_n represent the number of shares on day n . Write a recursive rule and an explicit formula. How many shares will the video receive on day 5?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

An arithmetic sequence has $a_3 = 18$ and $a_7 = 42$. Write a recursive rule and an explicit formula. Then find a_{12} and explain how you found the common difference.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	An arithmetic sequence has $a_1 = 5$ and common difference 6. Write a recursive rule and an explicit formula.	Recursive: $a_1 = 5, a_n = a_{(n-1)} + 6$ for $n \geq 2$. Explicit: $a_n = 5 + 6(n-1)$, or $a_n = 6n - 1$.
2	A geometric sequence has $b_1 = 2$ and common ratio 4. Write a recursive rule and an explicit formula.	Recursive: $b_1 = 2, b_n = 4b_{(n-1)}$ for $n \geq 2$. Explicit: $b_n = 2(4)^{(n-1)}$.
3	The explicit formula is $c_n = 9 - 2(n-1)$. Write a recursive rule. Then list the first four terms.	Recursive: $c_1 = 9, c_n = c_{(n-1)} - 2$ for $n \geq 2$. First four terms: 9, 7, 5, 3.
4	The explicit formula is $d_n = 3(5)^{(n-1)}$. Write a recursive rule and find d_4 .	Recursive: $d_1 = 3, d_n = 5d_{(n-1)}$ for $n \geq 2$. $d_4 = 375$.
5	The sequence begins 64, 32, 16, ... Write a recursive rule and an explicit formula.	Recursive: $a_1 = 64, a_n = (1/2)a_{(n-1)}$ for $n \geq 2$. Explicit: $a_n = 64(1/2)^{(n-1)}$.
6	A sequence is defined by $e_1 = -4$ and $e_n = e_{(n-1)} + 7$ for $n \geq 2$. Write an explicit formula and find e_6 .	Explicit: $e_n = -4 + 7(n-1)$, or $e_n = 7n - 11$. $e_6 = 31$.

PART 2 • APPLY IT

- Recursive:** $F_1 = 120, F_n = F_{(n-1)} + 35$ for $n \geq 2$. **Explicit:** $F_n = 120 + 35(n-1)$. **$F_6 = \$295$.** This is arithmetic because the fund increases by a constant \$35 each week. Substitute $n = 6$ into the explicit formula.
- Recursive:** $S_1 = 7, S_n = 2S_{(n-1)}$ for $n \geq 2$. **Explicit:** $S_n = 7(2)^{(n-1)}$. **$S_5 = 112$ shares.** This is geometric because the number of shares is multiplied by 2 each day. Substitute $n = 5$ into the explicit formula.

CHALLENGE

The common difference is $(42 - 18)/(7 - 3) = 6$. **Recursive:** $a_1 = 6, a_n = a_{(n-1)} + 6$ for $n \geq 2$. **Explicit:** $a_n = 6 + 6(n-1)$, or $a_n = 6n$. **$a_{12} = 72$.**

Accept equivalent explicit formulas, including expanded or factored forms. A recursive rule must include both the starting term and the condition $n \geq 2$.