One Way Only

Restrict a domain, then build an inverse.

HIGH SCHOOL · HSF-BF.B.4d

CCSS.Math.Content.HSF-BF.B.4d "(+) Produce an invertible function from a non-invertible function by restricting the domain."

NAME ______________________ DATE ______________

CHOOSE ONE BRANCH

A function has an inverse only when each output comes from exactly one input. For a U-shaped or absolute-value graph, restrict the domain to one branch, on one side of its turning point. Then solve y = f(x) for x using the branch chosen by the restriction, and swap x and y.

WORKED EXAMPLE

g(x) = (x-4)(x-4)+9 is not invertible on all real numbers. Restrict the domain to x >= 4 and find g^-1(x).

1. Let y = (x-4)(x-4)+9.
2. Subtract 9: y-9 = (x-4)(x-4).
3. Because x >= 4, x-4 is nonnegative, so x-4 = sqrt(y-9).
4. Solve for x and swap variables: g^-1(x) = 4+sqrt(x-9).

Answer: Restricted domain: x >= 4. g^-1(x) = 4+sqrt(x-9), with inverse domain x >= 9.

PRACTICE 6 problems

1. For f(x) = x^2, use the restriction x >= 0. Write f^-1(x) and state its domain.

ANSWER ______________

2. For h(x) = (x+1)^2, use the restriction x >= -1. Write h^-1(x) and state its domain.

ANSWER ______________

3. For p(x) = (x-3)^2+2, use the restriction x <= 3. Write p^-1(x) and state its domain.

ANSWER ______________

4. For q(x) = x^2-6, use the restriction x <= 0. Write q^-1(x) and state its domain.

ANSWER ______________

5. For r(x) = |x-5|, use the restriction x >= 5. Write r^-1(x) and state its domain.

ANSWER ______________

6. For s(x) = (2x+1)^2, use the restriction x >= -1/2. Write s^-1(x) and state its domain.

ANSWER ______________

APPLY IT
Show your work

1 A video editor models the brightness of a fading screen with $b(t) = -(t-3)^2+12$, where t is time in seconds and $0 \le t \le 6$. During the fade-out, use $3 \le t \le 6$. Write an inverse that gives time from brightness, and state the inverse domain.

WORK SPACE

ANSWER ______________________

2 In a game, an avatar's distance from a charging station is modeled by $d(t) = (t-6)^2+1$, where t is in seconds and $0 \le t \le 10$. After the avatar reaches its closest point, use $6 \le t \le 10$. Write the inverse, then find when the avatar is 5 blocks from the station.

WORK SPACE

ANSWER ______________________

CHALLENGE
One step up

CHALLENGE

For $f(x) = x^2-10x+11$, find both maximal interval domain restrictions that make f invertible. Write the inverse for each restriction, including its domain, and explain why the unrestricted function has no inverse.

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ANSWER KEY # One Way Only

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	1. For f(x) = x^2, use the restriction x >= 0. Write f^-1(x) and state its domain.	**f^-1(x) = sqrt(x), with domain x >= 0.**
2	2. For h(x) = (x+1)^2, use the restriction x >= -1. Write h^-1(x) and state its domain.	**h^-1(x) = -1+sqrt(x), with domain x >= 0.**
3	3. For p(x) = (x-3)^2+2, use the restriction x <= 3. Write p^-1(x) and state its domain.	**p^-1(x) = 3-sqrt(x-2), with domain x >= 2.**
4	4. For q(x) = x^2-6, use the restriction x <= 0. Write q^-1(x) and state its domain.	**q^-1(x) = -sqrt(x+6), with domain x >= -6.**
5	5. For r(x) = |x-5|, use the restriction x >= 5. Write r^-1(x) and state its domain.	**r^-1(x) = x+5, with domain x >= 0.**
6	6. For s(x) = (2x+1)^2, use the restriction x >= -1/2. Write s^-1(x) and state its domain.	**s^-1(x) = (sqrt(x)-1)/2, with domain x >= 0.**

PART 2 · APPLY IT

1. **b^-1(x) = 3+sqrt(12-x), with domain 3 <= x <= 12.** On the fade-out interval, t-3 is nonnegative, so solve for the positive square-root branch. The brightness ranges from 12 down to 3 on this interval.
2. **d^-1(x) = 6+sqrt(x-1), with domain 1 <= x <= 17. The avatar is 5 blocks away at t = 8 seconds.** Use the branch t >= 6, so the inverse uses the positive square root. Substituting 5 gives d^-1(5) = 6+2 = 8, and d(8) = 5.

CHALLENGE

Since f(x) = (x-5)^2-14, the two maximal interval restrictions are x >= 5 and x <= 5. For x >= 5, f^-1(x) = 5+sqrt(x+14), with domain x >= -14. For x <= 5, f^-1(x) = 5-sqrt(x+14), with domain x >= -14. Without a restriction, inputs equally far from 5 have the same output, so one output can come from two inputs.

Accept equivalent inverse notation and equivalent domain statements. For the challenge, students must give both branch restrictions and use the matching positive or negative square-root branch.