LESSON MAKER PRO

One Branch Only

Restrict a function so its inverse works.

HIGH SCHOOL · HSF-BF.B.4d

CCSS.Math.Content.HSF-BF.B.4d "(+) Produce an invertible function from a non-invertible function by restricting the domain."

NAME _____ DATE _____

CHOOSE ONE SIDE

An inverse function must match each output with only one input. A U-shaped parabola is not one-to-one across both sides of its vertex, so restrict its domain to one side of the vertex, including the vertex.

WORKED EXAMPLE

Let $f(x) = (x - 7)^2 + 5$. Restrict the domain to $x >= 7$. Find $f^{-1}(x)$.

1. Switch the input and output: $x = (f^{-1}(x) - 7)^2 + 5$.
2. Subtract 5: $x - 5 = (f^{-1}(x) - 7)^2$.
3. Use the positive square root because the restricted inputs are at least 7: $f^{-1}(x) - 7 = \text{sqrt}(x - 5)$.

Answer: $f^{-1}(x) = 7 + \text{sqrt}(x - 5)$, with domain $x >= 5$.

PRACTICE 6 problems

1 For $f(x) = x^2 - 4$, restrict the domain to $x >= 0$. Write $f^{-1}(x)$ and the domain of the inverse.

ANSWER _____

2 For $g(x) = (x - 3)^2$, restrict the domain to $x <= 3$. Write $g^{-1}(x)$ and the domain of the inverse.

ANSWER _____

3 For $h(x) = |x + 2|$, restrict the domain to $x >= -2$. Write $h^{-1}(x)$ and the domain of the inverse.

ANSWER _____

4 For $p(x) = x^2 + 6x$, restrict the domain to $x <= -3$. Write $p^{-1}(x)$ and the domain of the inverse.

ANSWER _____

5 For $q(x) = (x + 1)^2 - 8$, restrict the domain to $x <= -1$. Write $q^{-1}(x)$ and the domain of the inverse.

ANSWER _____

6 The function $r(x) = x^2 - 1$ has $r(-2) = r(2)$. Explain in one sentence why r is not invertible on all real numbers, then give one domain restriction that makes it invertible.

ANSWER _____

APPLY IT Show your work

1 A basketball's height is modeled by $h(x) = -x^2 + 8x + 1$, where x is the number of seconds after launch. A coach wants each height to match exactly one time while the ball is rising. State the restricted domain, write $h^{-1}(x)$, and state the domain of the inverse.

WORK SPACE

ANSWER _____

2 A student council estimates net profit with $P(x) = -x^2 + 12x$, where x is the ticket price in dollars and $0 <= x <= 12$. To identify a unique low ticket price from a profit amount, restrict the domain to $0 <= x <= 6$. Write $P^{-1}(x)$, then find the ticket price when the profit is $27.

WORK SPACE

ANSWER _____

CHALLENGE One step up

For $f(x) = x^2 - 12x + 20$, use a restriction of the form $x >= a$ that includes the vertex and makes f invertible. Find a, $f^{-1}(x)$, and the domain of the inverse. Explain why the positive square root is needed.

WORK SPACE AND ANSWER

One Branch Only

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For $f(x) = x^2 - 4$, restrict the domain to $x >= 0$. Write $f^{-1}(x)$ and the domain of the inverse.	**$f^{-1}(x) = \text{sqrt}(x + 4)$; domain: $x >= -4$**
2	For $g(x) = (x - 3)^2$, restrict the domain to $x <= 3$. Write $g^{-1}(x)$ and the domain of the inverse.	**$g^{-1}(x) = 3 - \text{sqrt}(x)$; domain: $x >= 0$**
3	For $h(x) = \lvert x + 2 \rvert$, restrict the domain to $x >= -2$. Write $h^{-1}(x)$ and the domain of the inverse.	**$h^{-1}(x) = x - 2$; domain: $x >= 0$**
4	For $p(x) = x^2 + 6x$, restrict the domain to $x <= -3$. Write $p^{-1}(x)$ and the domain of the inverse.	**$p^{-1}(x) = -3 - \text{sqrt}(x + 9)$; domain: $x >= -9$**
5	For $q(x) = (x + 1)^2 - 8$, restrict the domain to $x <= -1$. Write $q^{-1}(x)$ and the domain of the inverse.	**$q^{-1}(x) = -1 - \text{sqrt}(x + 8)$; domain: $x >= -8$**
6	The function $r(x) = x^2 - 1$ has $r(-2) = r(2)$. Explain in one sentence why r is not invertible on all real numbers, then give one domain restriction that makes it invertible.	**Both -2 and 2 give output 3, so r is not one-to-one; restrict the domain to $x >= 0$.**

PART 2 · APPLY IT

1. **Restrict h to $0 <= x <= 4$. $h^{-1}(x) = 4 - \text{sqrt}(17 - x)$, with domain $1 <= x <= 17$.** Rewrite the model as $h(x) = 17 - (x - 4)^2$. The rising branch uses inputs from 0 through 4, so select the negative square-root branch when solving for the original input.
2. **$P^{-1}(x) = 6 - \text{sqrt}(36 - x)$, with domain $0 <= x <= 36$. $P^{-1}(27) = 3$, so the ticket price is $3.** Rewrite the profit model as $P(x) = 36 - (x - 6)^2$. The low-price restriction requires the branch with original inputs at most 6.

CHALLENGE

$a = 6$; $f^{-1}(x) = 6 + \text{sqrt}(x + 16)$, with domain $x >= -16$. The positive square root is needed because the restricted original inputs must be at least 6.

Accept equivalent inverse notation and equivalent domain statements. In Problem 6, $x <= 0$ is also a valid restriction because it selects the other one-to-one branch.