

Intercepts and Peaks

Graph lines and parabolas using their key points.

HIGH SCHOOL • HSF-IF.C.7a

CCSS.Math.Content.HSF-IF.C.7a "Graph linear and quadratic functions and show intercepts, maxima, and minima."

NAME _____

DATE _____

GRAPHING KEY FEATURES

For a line, find the x-intercept by setting y to 0 and the y-intercept by setting x to 0. For a quadratic $y = ax^2 + bx + c$, find intercepts the same way, then find the vertex using $x = -b/(2a)$. A positive a gives a minimum, and a negative a gives a maximum.

WORKED EXAMPLE

Graph $y = (x - 9)^2 - 4$. Label the intercepts and the vertex.

1. The vertex is $(9, -4)$, because the function is in vertex form.
2. The coefficient of the squared term is positive, so the vertex is a minimum.
3. Set $y = 0$: $(x - 9)^2 - 4 = 0$, so $x = 7$ or $x = 11$.
4. Set $x = 0$: $y = (-9)^2 - 4 = 77$.

Answer: Vertex and minimum: $(9, -4)$. x-intercepts: $(7, 0)$ and $(11, 0)$. y-intercept: $(0, 77)$.

PRACTICE 6 problems

- 1** For $y = 3x - 8$, find both intercepts. Then sketch the line through them.

ANSWER _____

- 2** For $y = -5x + 12$, find both intercepts. Then sketch the line through them.

ANSWER _____

- 3** Graph $y = x^2 + 2x - 15$. Label the x-intercepts, y-intercept, vertex, and minimum or maximum.

ANSWER _____

- 4** Graph $y = -x^2 + 12x - 20$. Label the x-intercepts, y-intercept, vertex, and minimum or maximum.

ANSWER _____

- 5** Graph $y = (x + 3)^2 - 25$. Label the x-intercepts, y-intercept, vertex, and minimum or maximum.

ANSWER _____

- 6** Graph $y = -3(x - 2)^2 + 27$. Label the x-intercepts, y-intercept, vertex, and minimum or maximum.

ANSWER _____

APPLY IT Show your work

- 1** A video game design club models the height of a launched game object with $h(t) = -t^2 + 12t + 13$, where t is time in seconds and h is height in meters. Graph the function for $t \geq 0$. Label the starting height, the maximum height, and the time when the object reaches the ground.

WORK SPACE

ANSWER _____

- 2** A student council models the profit from a concert ticket price with $P(x) = -x^2 + 20x - 75$, where x is the ticket price in dollars and P is profit in hundreds of dollars. Graph the function for $x \geq 0$. At which ticket prices does the concert break even, and what price gives the greatest profit? Include the y-intercept on your graph.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A quadratic has x-intercepts at $(-2, 0)$ and $(6, 0)$, and it passes through $(0, -8)$. Find an equation for the quadratic. Then find its vertex and state whether the vertex is a maximum or minimum. Explain how the intercepts and the point $(0, -8)$ determine your equation.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	For $y = 3x - 8$, find both intercepts. Then sketch the line through them.	x-intercept: $(8/3, 0)$. y-intercept: $(0, -8)$.
2	For $y = -5x + 12$, find both intercepts. Then sketch the line through them.	x-intercept: $(12/5, 0)$. y-intercept: $(0, 12)$.
3	Graph $y = x^2 + 2x - 15$. Label the x-intercepts, y-intercept, vertex, and minimum or maximum.	x-intercepts: $(-5, 0)$ and $(3, 0)$. y-intercept: $(0, -15)$. Vertex and minimum: $(-1, -16)$.
4	Graph $y = -x^2 + 12x - 20$. Label the x-intercepts, y-intercept, vertex, and minimum or maximum.	x-intercepts: $(2, 0)$ and $(10, 0)$. y-intercept: $(0, -20)$. Vertex and maximum: $(6, 16)$.
5	Graph $y = (x + 3)^2 - 25$. Label the x-intercepts, y-intercept, vertex, and minimum or maximum.	x-intercepts: $(-8, 0)$ and $(2, 0)$. y-intercept: $(0, -16)$. Vertex and minimum: $(-3, -25)$.
6	Graph $y = -3(x - 2)^2 + 27$. Label the x-intercepts, y-intercept, vertex, and minimum or maximum.	x-intercepts: $(-1, 0)$ and $(5, 0)$. y-intercept: $(0, 15)$. Vertex and maximum: $(2, 27)$.

PART 2 • APPLY IT

- Starting height: $(0, 13)$. Maximum: $(6, 49)$. The object reaches the ground at $(13, 0)$, so it lands after 13 seconds.** Use $h(0)$ for the starting height. Factor or solve $h(t) = 0$ for the ground time, then use the vertex at $t = 6$ for the maximum.
- Break-even ticket prices: \$5 and \$15, at $(5, 0)$ and $(15, 0)$. Greatest profit: 25 hundreds of dollars, or \$2,500, at a ticket price of \$10, shown by $(10, 25)$. y-intercept: $(0, -75)$.** Set $P(x) = 0$ to find the break-even prices. The vertex is at $x = 10$ and gives $P(10) = 25$.

CHALLENGE

$y = (2/3)(x + 2)(x - 6)$, or $y = (2/3)x^2 - (8/3)x - 8$. The vertex is $(2, -32/3)$, and it is a minimum. The intercepts give $y = a(x + 2)(x - 6)$. Substituting $(0, -8)$ gives $-12a = -8$, so $a = 2/3$.

Accept equivalent factored, standard, or vertex forms for quadratic equations. For sketches, accept graphs with correctly placed intercepts and vertices, correct opening direction, and reasonable scale.