

Quadratic Clues

Reveal zeros, peaks, and symmetry from quadratic forms.

HIGH SCHOOL • HSF-IF.C.8a

CCSS.Math.Content.HSF-IF.C.8a "Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context."

NAME _____

DATE _____

READ THE FUNCTION FORMS

Factored form, $a(x-r)(x-s)$, shows the zeros r and s . The line of symmetry is halfway between the zeros. Vertex form, $a(x-h)^2+k$, shows the extreme value k at $x=h$, with a minimum when a is positive and a maximum when a is negative.

WORKED EXAMPLE

For $f(x)=x^2-18x+77$, find the zeros, line of symmetry, and extreme value.

1. Find two numbers with product 77 and sum -18: -7 and -11.
2. Factor: $f(x)=(x-7)(x-11)$, so the zeros are $x=7$ and $x=11$.
3. Average the zeros: $(7+11)/2=9$, so the line of symmetry is $x=9$.
4. Complete the square: $f(x)=(x-9)^2-4$, so the minimum value is -4.

Answer: Zeros: $x=7$ and $x=11$. Line of symmetry: $x=9$. Minimum: -4 at $x=9$.

PRACTICE 6 problems

- 1** Factor $f(x)=x^2-13x+30$. State the zeros and the line of symmetry.

ANSWER _____

- 2** Complete the square for $g(x)=x^2+6x-15$. State the vertex and minimum value.

ANSWER _____

- 3** Use factoring to find the zeros, line of symmetry, and maximum of $h(x)=-x^2+12x-20$.

ANSWER _____

- 4** Write $p(x)=2x^2-12x+10$ in vertex form and factored form. Then state its zeros and minimum.

ANSWER _____

- 5** For $q(x)=-(x+5)^2+14$, state the vertex, line of symmetry, and maximum value.

ANSWER _____

- 6** Find the zeros, line of symmetry, and minimum of $r(x)=x^2-2x-15$.

ANSWER _____

APPLY IT Show your work

- 1** At a school event, the height of a launched foam ball is modeled by $h(t) = -2t^2 + 8t + 24$, where h is height in feet and t is time in seconds. When does the ball hit the floor? What is its maximum height, and when does it occur? State the line of symmetry.

WORK SPACE

ANSWER _____

- 2** A student club models its net profit from an event by $P(x) = -20x^2 + 120x - 100$, where P is profit in dollars and x is the entry fee in dollars. Find the entry fees that make the club break even. What entry fee gives the greatest profit, and what is that profit?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A quadratic function has zeros at $x=1$ and $x=5$, and its minimum value is -20 . Write the function in factored form, vertex form, and standard form. State its line of symmetry.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Factor $f(x)=x^2-13x+30$. State the zeros and the line of symmetry.	$f(x)=(x-3)(x-10)$; zeros: $x=3$ and $x=10$; line of symmetry: $x=13/2$.
2	Complete the square for $g(x)=x^2+6x-15$. State the vertex and minimum value.	$g(x)=(x+3)^2-24$; vertex: $(-3,-24)$; minimum value: -24.
3	Use factoring to find the zeros, line of symmetry, and maximum of $h(x)=-x^2+12x-20$.	$h(x)=-(x-2)(x-10)$; zeros: $x=2$ and $x=10$; line of symmetry: $x=6$; maximum: 16.
4	Write $p(x)=2x^2-12x+10$ in vertex form and factored form. Then state its zeros and minimum.	Vertex form: $p(x)=2(x-3)^2-8$. Factored form: $p(x)=2(x-1)(x-5)$. Zeros: $x=1$ and $x=5$; minimum: -8.
5	For $q(x)=-(x+5)^2+14$, state the vertex, line of symmetry, and maximum value.	Vertex: $(-5,14)$; line of symmetry: $x=-5$; maximum value: 14.
6	Find the zeros, line of symmetry, and minimum of $r(x)=x^2-2x-15$.	$r(x)=(x-5)(x+3)=(x-1)^2-16$; zeros: $x=-3$ and $x=5$; line of symmetry: $x=1$; minimum: -16.

PART 2 • APPLY IT

- The ball hits the floor at $t=6$ seconds. Its maximum height is 32 feet at $t=2$ seconds. The line of symmetry is $t=2$.** Factor $h(t)=-2(t-6)(t+2)$, so the meaningful zero is $t=6$. Complete the square or use the midpoint of the zeros to find the vertex $(2,32)$.
- The club breaks even when $x=\$1$ or $x=\$5$. The greatest profit is \$80 when the entry fee is \$3.** Factor $P(x)=-20(x-1)(x-5)$ to find the break-even fees. The axis halfway between 1 and 5 is $x=3$, and $P(3)=80$.

CHALLENGE

Factored form: $y=5(x-1)(x-5)$. Vertex form: $y=5(x-3)^2-20$. Standard form: $y=5x^2-30x+25$. Line of symmetry: $x=3$.

Accept equivalent factored, vertex, or standard forms when they expand to the same function. For contextual questions, accept only nonnegative time values and entry fees as meaningful interpretations.