

Change Pattern Detectives

Use equal intervals to spot additive and multiplicative growth.

HIGH SCHOOL • HSF-LE.A.1a

CCSS.Math.Content.HSF-LE.A.1a "Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals."

NAME _____

DATE _____

EQUAL INTERVALS, CLEAR PATTERNS

When x changes by equal amounts, a linear function adds or subtracts the same amount from its output each time. When x changes by equal amounts, an exponential function multiplies its output by the same factor each time.

WORKED EXAMPLE

The table shows a function. Decide whether it is linear or exponential:
 x : 10, 11, 12; y : 7, 28, 112.

1. The x -values increase by 1 each time.
2. $28 / 7 = 4$ and $112 / 28 = 4$.
3. The y -values are multiplied by 4 over each equal x -interval.

Answer: Exponential, with a common factor of 4 for each increase of 1 in x .

PRACTICE 6 problems

- 1** Classify the table as linear or exponential. State the repeated change or factor. x : -1, 1, 3; y : -3, 5, 13.

ANSWER _____

- 2** For $f(x) = 6x + 1$, compare $f(2)$, $f(5)$, and $f(8)$. Is the function linear or exponential? Use the output changes to support your answer.

ANSWER _____

- 3** Classify the table as linear or exponential. State the repeated change or factor. x : -1, 0; y : 81, 27, 9.

ANSWER _____

- 4** For $g(x) = 3^x$, compare $g(1)$, $g(3)$, and $g(5)$. What common factor occurs when x increases by 2?

ANSWER _____

- 5** A sequence has values 18, 13, 8, 3 at equally spaced x -values. Is it linear or exponential? Explain with the repeated change or factor.

ANSWER _____

- 6** Classify the table as linear or exponential. State the repeated change or factor. x : 2, 5, 8; y : 2, 18, 162.

ANSWER _____

APPLY IT Show your work

- 1** A student starts a playlist with 19 songs and adds 6 songs each week. Let $L(w)$ be the number of songs after w weeks. Is this relationship linear or exponential? Write a rule for $L(w)$ and find $L(3)$.

WORK SPACE

ANSWER _____

- 2** A video club posts a clip that has 8 views on day 0. The number of views triples each day. Let $V(d)$ be the number of views after d days. Is this relationship linear or exponential? Write a rule for $V(d)$ and find $V(3)$.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Two tables use equal x -intervals. Table A: x : -3, 0, 3; y : 5, 25, 125. Table B: x : -3, 0, 3; y : 5, 19, 33. Which table is exponential and which is linear? Explain your reasoning using equal intervals.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Classify the table as linear or exponential. State the repeated change or factor. x: -1, 1, 3; y: -3, 5, 13.	Linear; the outputs increase by 8 for each increase of 2 in x.
2	For $f(x) = 6x + 1$, compare $f(2)$, $f(5)$, and $f(8)$. Is the function linear or exponential? Use the output changes to support your answer.	$f(2) = 13$, $f(5) = 31$, and $f(8) = 49$. The function is linear because the outputs increase by 18 when x increases by 3.
3	Classify the table as linear or exponential. State the repeated change or factor. x: -2, -1, 0; y: 81, 27, 9.	Exponential; the outputs are multiplied by $1/3$ for each increase of 1 in x.
4	For $g(x) = 3^x$, compare $g(1)$, $g(3)$, and $g(5)$. What common factor occurs when x increases by 2?	$g(1) = 3$, $g(3) = 27$, and $g(5) = 243$. The common factor is 9.
5	A sequence has values 18, 13, 8, 3 at equally spaced x-values. Is it linear or exponential? Explain with the repeated change or factor.	Linear; each value decreases by 5.
6	Classify the table as linear or exponential. State the repeated change or factor. x: 2, 5, 8; y: 2, 18, 162.	Exponential; the outputs are multiplied by 9 for each increase of 3 in x.

PART 2 • APPLY IT

- Linear. $L(w) = 19 + 6w$, and $L(3) = 37$ songs.** The playlist grows by the same additive amount, 6 songs, each week. Substitute 3 for w in $19 + 6w$.
- Exponential. $V(d) = 8 \times 3^d$, and $V(3) = 216$ views.** The number of views is multiplied by 3 each day. Substitute 3 for d in 8×3^d .

CHALLENGE

Table A is exponential because its outputs are multiplied by 5 over each increase of 3 in x. Table B is linear because its outputs increase by 14 over each increase of 3 in x.

Accept equivalent statements of constant difference or constant factor over equal x-intervals. For exponential tables that decrease, accept a reciprocal factor such as $1/3$.