

Growth Takes Over

Use tables to compare linear, quadratic, cubic, and exponential growth.

HIGH SCHOOL • HSF-LE.A.3

CCSS.Math.Content.HSF-LE.A.3 "Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function."

NAME _____

DATE _____

THE BIG IDEA

A linear or polynomial quantity may be larger at first, but an exponential quantity is multiplied by the same factor again and again. Make a table with increasing x -values to see when the exponential values become greater, then check the earlier values to find the first crossing point.

WORKED EXAMPLE

Find the first whole number $x \geq 0$ for which $E(x) = 17^x$ is greater than $L(x) = 300x + 400$.

- At $x = 0, 1$, and 2 , $E(x)$ is $1, 17$, and 289 , while $L(x)$ is $400, 700$, and 1000 .
- At $x = 3$, $E(3) = 17^3 = 4913$.
- At $x = 3$, $L(3) = 300(3) + 400 = 1300$.
- Since $4913 > 1300$ and $E(x)$ was smaller for $x = 0, 1$, and 2 , the first crossing is at $x = 3$.

Answer: $x = 3$

PRACTICE 6 problems

- 1** Use the table. At what value of x does $E(x)$ first become greater than $L(x)$? $x: 0 \ 1 \ 2 \ 3 \ 4 \ 5$
 $L(x): 4 \ 13 \ 22 \ 31 \ 40 \ 49 \ 58$ $E(x): 1 \ 2 \ 4 \ 8 \ 16 \ 32 \ 64$

ANSWER _____

- 2** At $x = 4$, compare $Q(x) = 3x^2 + 2$ and $E(x) = 4^x$. Which quantity is greater, and by how much?

ANSWER _____

- 3** Find the first whole number $x \geq 0$ for which $3^x > x^2 + 20$.

ANSWER _____

- 4** Complete the $E(x)$ row, where $L(x) = 8x + 1$ and $E(x) = 2^x$. Then state the first x for which $E(x) > L(x)$. $x: 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6$ $L(x): 1 \ 9 \ 17 \ 25 \ 33 \ 41 \ 49$ $E(x): _ _ _ _ _ _$

ANSWER _____

- 5** For which listed value of x is $E(x) = 2^x$ greater than $P(x) = x^3 + 10x$? $x = 8, x = 10$, or $x = 11$

ANSWER _____

- 6** At $x = 5$, compare $A(x) = 6x^2$ and $B(x) = 3^x$. Which quantity is greater, and by how much?

ANSWER _____

APPLY IT Show your work

- 1** A student account has 40 followers and gains 12 followers each day. A short-video account starts with 3 followers, and its followers double each day. Let d be the number of days after today. Use a table for $d = 0$ through $d = 6$. On what day does the short-video account first have more followers?

WORK SPACE

ANSWER _____

- 2** In a game tournament, Skill Path A gives a total of $5r^2$ points after r rounds. Skill Path B gives $2^{(r+1)}$ total points after r rounds. Use a table for $r = 1$ through $r = 7$. At what round does Skill Path B first give more total points?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Make a table for $n = 3$ through $n = 10$ to compare 2^n and n^3 . What is the first value of n in this range for which $2^n > n^3$? Explain how your table shows it is the first.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Use the table. At what value of x does $E(x)$ first become greater than $L(x)$? x : 0 1 2 3 4 5 6 $L(x)$: 4 13 22 31 40 49 58 $E(x)$: 1 2 4 8 16 32 64	$x = 6$
2	At $x = 4$, compare $Q(x) = 3x^2 + 2$ and $E(x) = 4^x$. Which quantity is greater, and by how much?	$E(x)$ is greater by 206. $Q(4) = 50$ and $E(4) = 256$.
3	Find the first whole number $x \geq 0$ for which $3^x > x^2 + 20$.	$x = 4$
4	Complete the $E(x)$ row, where $L(x) = 8x + 1$ and $E(x) = 2^x$. Then state the first x for which $E(x) > L(x)$. x : 0 1 2 3 4 5 6 $L(x)$: 1 9 17 25 33 41 49 $E(x)$: _____	$E(x)$: 1, 2, 4, 8, 16, 32, 64; first $x = 6$
5	For which listed value of x is $E(x) = 2^x$ greater than $P(x) = x^3 + 10x$? $x = 8$, $x = 10$, or $x = 11$	$x = 11$. $E(11) = 2048$ and $P(11) = 1441$.
6	At $x = 5$, compare $A(x) = 6x^2$ and $B(x) = 3^x$. Which quantity is greater, and by how much?	$B(x)$ is greater by 93. $A(5) = 150$ and $B(5) = 243$.

PART 2 • APPLY IT

- Day 6. The short-video account has 192 followers, and the student account has 112 followers.** Compare $S(d) = 40 + 12d$ with $V(d) = 3(2^d)$. At $d = 5$, $V(5) = 96 < 100$, and at $d = 6$, $V(6) = 192 > 112$.
- Round 7. Skill Path B gives 256 points, and Skill Path A gives 245 points.** Compare $A(r) = 5r^2$ with $B(r) = 2^{(r+1)}$. At $r = 6$, $128 < 180$, and at $r = 7$, $256 > 245$.

CHALLENGE

$n = 10$. For $n = 3$ through 9, $2^n < n^3$: $8 < 27$, $16 < 64$, $32 < 125$, $64 < 216$, $128 < 343$, $256 < 512$, and $512 < 729$. At $n = 10$, $1024 > 1000$.

Accept equivalent comparison statements and correctly completed tables. For first-crossing questions, students should show or state that the exponential value was not greater at the previous whole-number input.