LESSON MAKER PRO

Circle Beyond One Turn

Use radian angles to locate points on the unit circle.

HIGH SCHOOL · HSF-TF.A.2

CCSS.Math.Content.HSF-TF.A.2 "Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle."

NAME ______________________ DATE ____________

ONE CIRCLE, ALL ANGLES

An angle theta starts at (1, 0) on the unit circle, and its terminal point is (cos(theta), sin(theta)). Adding or subtracting 2pi radians makes a full trip around the circle and leaves the point unchanged, so angles greater than 2pi or less than 0 still have sine and cosine values.

WORKED EXAMPLE

Find the terminal point and the values of cos(11pi/6) and sin(11pi/6).

1. 11pi/6 is 2pi - pi/6, so the terminal side is in Quadrant IV.
2. The unit-circle point for pi/6 is (sqrt(3)/2, 1/2).
3. In Quadrant IV, the x-coordinate stays positive and the y-coordinate is negative: (sqrt(3)/2, -1/2).

Answer: Terminal point: (sqrt(3)/2, -1/2); cos(11pi/6) = sqrt(3)/2; sin(11pi/6) = -1/2.

PRACTICE

6 problems

1. Starting at (1, 0), move counterclockwise pi/3 radians. What is the terminal point?

   ANSWER ____________

2. Find cos(5pi/4) and sin(5pi/4).

   ANSWER ____________

3. Starting at (1, 0), traverse -pi/2 radians. What is the terminal point?

   ANSWER ____________

4. What is the terminal point for an angle of 7pi/4 radians?

   ANSWER ____________

5. What is the terminal point for an angle of 2pi radians?

   ANSWER ____________

6. What is the terminal point for an angle of 10pi/3 radians?

   ANSWER ____________

APPLY IT
Show your work

1 During a coding-club game, a character's direction is shown by a point moving on a unit-circle compass. It starts at (1, 0) and turns counterclockwise 17pi/4 radians. What is its final coordinate?

WORK SPACE

ANSWER ______________________

2 A music-video editor uses a unit-circle dial for a spinning marker. Starting at (1, 0), the marker's turn is recorded as -4pi/3 radians. What is the marker's final coordinate?

WORK SPACE

ANSWER ______________________

CHALLENGE
One step up

CHALLENGE

A student says that an angle must be between 0 and 2pi to have a sine and cosine. Use theta = -21pi/4 to refute the claim. Find a coterminal angle between 0 and 2pi, give the terminal point, and explain why the original angle has the same point.

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ANSWER KEY

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Starting at (1, 0), move counterclockwise pi/3 radians. What is the terminal point?	**(1/2, sqrt(3)/2)**
2	Find cos(5pi/4) and sin(5pi/4).	**cos(5pi/4) = -sqrt(2)/2; sin(5pi/4) = -sqrt(2)/2**
3	Starting at (1, 0), traverse -pi/2 radians. What is the terminal point?	**(0, -1)**
4	What is the terminal point for an angle of 7pi/4 radians?	**(sqrt(2)/2, -sqrt(2)/2)**
5	What is the terminal point for an angle of 2pi radians?	**(1, 0)**
6	What is the terminal point for an angle of 10pi/3 radians?	**(-1/2, -sqrt(3)/2)**

PART 2 · APPLY IT

1. **(sqrt(2)/2, sqrt(2)/2)** Subtract 4pi, or 16pi/4, from 17pi/4 to get pi/4. The unit-circle point at pi/4 is (sqrt(2)/2, sqrt(2)/2).
2. **(-1/2, sqrt(3)/2)** Add 2pi, or 6pi/3, to -4pi/3 to get 2pi/3. The unit-circle point at 2pi/3 is (-1/2, sqrt(3)/2).

CHALLENGE

-21pi/4 + 3(2pi) = 3pi/4, so a coterminal angle is 3pi/4. Its terminal point is (-sqrt(2)/2, sqrt(2)/2). Adding 2pi three times makes three full trips around the unit circle, so the terminal point does not change.

Accept equivalent radical formatting and correctly ordered coordinate pairs. For the challenge, accept any correct explanation that adding whole multiples of 2pi leaves the terminal point unchanged.