

Triangle Turnaround

Special-angle values and unit-circle transformations.

HIGH SCHOOL • HSF-TF.A.3

CCSS.Math.Content.HSF-TF.A.3 “(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x , $\pi + x$, and $2\pi - x$.”

NAME _____

DATE _____

COORDINATES GUIDE YOU

Use 30-60-90 and 45-45-90 triangle ratios to find coordinates on the unit circle for special angles. On the unit circle, cosine is the x-coordinate, sine is the y-coordinate, and tangent is y/x . Adding π moves to the opposite point, while $2\pi - x$ reflects a point across the x-axis.

WORKED EXAMPLE

Find $\tan(7\pi/4)$.

- $7\pi/4$ is in Quadrant IV, and its reference angle is $\pi/4$.
- The unit-circle coordinates are $(\sqrt{2}/2, -\sqrt{2}/2)$.
- $\tan(7\pi/4) = y/x = (-\sqrt{2}/2)/(\sqrt{2}/2) = -1$.

Answer: $\tan(7\pi/4) = -1$

PRACTICE 6 problems

- 1** Find $\sin(\pi/6)$, $\cos(\pi/6)$, and $\tan(\pi/6)$, in that order.

ANSWER _____

- 2** Find $\sin(\pi/4)$, $\cos(\pi/4)$, and $\tan(\pi/4)$, in that order.

ANSWER _____

- 3** Find $\sin(\pi/3)$, $\cos(\pi/3)$, and $\tan(\pi/3)$, in that order.

ANSWER _____

- 4** If $\sin x = a$, $\cos x = b$, and $\tan x = c$, express $\sin(\pi + x)$, $\cos(\pi + x)$, and $\tan(\pi + x)$, in that order.

ANSWER _____

- 5** If $\sin x = a$, $\cos x = b$, and $\tan x = c$, express $\sin(2\pi - x)$, $\cos(2\pi - x)$, and $\tan(2\pi - x)$, in that order.

ANSWER _____

- 6** If $\sin x = 5/13$ and $\cos x = -12/13$, find $\sin(2\pi - x)$, $\cos(2\pi - x)$, and $\tan(2\pi - x)$, in that order.

ANSWER _____

APPLY IT Show your work

- 1** A robotics team programs a rover to travel 8 m up a straight ramp. The ramp makes an angle of $\pi/6$ with level ground. How much vertical height does the rover gain?

WORK SPACE

ANSWER _____

- 2** For a student news video, a camera is 10 m from a banner along a line that makes an angle of $\pi/4$ with the horizontal. How far away is the banner horizontally?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

For any x where tangent is defined, a student says that $\tan(2\pi - x) = -\tan x$. Is the student correct? Explain using sine and cosine.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Find $\sin(\pi/6)$, $\cos(\pi/6)$, and $\tan(\pi/6)$, in that order.	(1/2, $\sqrt{3}/2$, $\sqrt{3}/3$)
2	Find $\sin(\pi/4)$, $\cos(\pi/4)$, and $\tan(\pi/4)$, in that order.	($\sqrt{2}/2$, $\sqrt{2}/2$, 1)
3	Find $\sin(\pi/3)$, $\cos(\pi/3)$, and $\tan(\pi/3)$, in that order.	($\sqrt{3}/2$, 1/2, $\sqrt{3}$)
4	If $\sin x = a$, $\cos x = b$, and $\tan x = c$, express $\sin(\pi + x)$, $\cos(\pi + x)$, and $\tan(\pi + x)$, in that order.	(-a, -b, c)
5	If $\sin x = a$, $\cos x = b$, and $\tan x = c$, express $\sin(2\pi - x)$, $\cos(2\pi - x)$, and $\tan(2\pi - x)$, in that order.	(-a, b, -c)
6	If $\sin x = 5/13$ and $\cos x = -12/13$, find $\sin(2\pi - x)$, $\cos(2\pi - x)$, and $\tan(2\pi - x)$, in that order.	(-5/13, -12/13, 5/12)

PART 2 • APPLY IT

- 4 m** The vertical height is the opposite side of the right triangle. Compute $8\sin(\pi/6) = 8(1/2) = 4$ m.
- 5 $\sqrt{2}$ m** The horizontal distance is the adjacent side. Compute $10\cos(\pi/4) = 10(\sqrt{2}/2) = 5\sqrt{2}$ m.

CHALLENGE

Yes. $\sin(2\pi - x) = -\sin x$ and $\cos(2\pi - x) = \cos x$, so $\tan(2\pi - x) = (-\sin x)/(\cos x) = -\tan x$.

Accept equivalent radical forms, including $1/\sqrt{3}$ for $\sqrt{3}/3$. For ordered triples, require the values in the order requested.