LESSON MAKER PRO

Triangle Turn Rules

Exact trig values from special triangles and unit-circle turns

HIGH SCHOOL · HSF-TF.A.3

CCSS.Math.Content.HSF-TF.A.3 "(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for x, π + x, and 2&pi..."

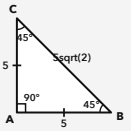
NAME _____ DATE _____

THE BIG IDEA

Special right triangles give exact side ratios, so you can find trig values without decimals. On the unit circle, a half-turn reverses both coordinates, while turning to 2pi - x reverses only the vertical coordinate.

WORKED EXAMPLE

Use triangle ABC to find sin(pi/4), cos(pi/4), and tan(pi/4).

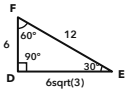


1. The right angle is at A, so BC is the hypotenuse and has length 5sqrt(2).
2. For angle B, AC = 5 is opposite and AB = 5 is the adjacent leg.
3. sin(pi/4) = 5/(5sqrt(2)) = sqrt(2)/2, cos(pi/4) = 5/(5sqrt(2)) = sqrt(2)/2, and tan(pi/4) = 5/5 = 1.

Answer: sin(pi/4) = sqrt(2)/2, cos(pi/4) = sqrt(2)/2, tan(pi/4) = 1

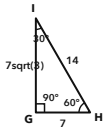
PRACTICE 6 problems

1 Use triangle DEF to find sin(pi/6), cos(pi/6), and tan(pi/6).



ANSWER _____

2 Use triangle GHI to find sin(pi/3), cos(pi/3), and tan(pi/3).



ANSWER _____

3 Suppose sin(x) = -3/5 and cos(x) = 4/5. Find sin(pi + x), cos(pi + x), and tan(pi + x).

ANSWER _____

4 Suppose sin(x) = 5/13 and cos(x) = -12/13. Find sin(2pi - x), cos(2pi - x), and tan(2pi - x).

ANSWER _____

5 At angle x, cos(x) = 7/25 and sin(x) = 24/25. Find cos(pi + x) and sin(pi + x).

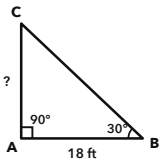
ANSWER _____

6 At angle x, sin(x) = -8/17 and cos(x) = -15/17. Find tan(2pi - x).

ANSWER _____

APPLY IT Show your work

1 A skate park ramp forms right triangle ABC shown. The ground segment AB is 18 feet long, and the angle at B is pi/6. Find the vertical rise AC in simplest radical form.



ANSWER _____

2 A student coding a game uses the point (cos(theta), sin(theta)) to place a sprite around a circle. At theta = x, the sprite is at (-15/17, 8/17). What are the sprite's coordinates at theta = pi + x and theta = 2pi - x?

WORK SPACE

ANSWER _____

CHALLENGE One step up

For every x where tangent is defined, explain why tan(pi + x) + tan(2pi - x) = 0. Use sine and cosine relationships in your explanation.

WORK SPACE AND ANSWER

ANSWER KEY

Triangle Turn Rules

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Use triangle DEF to find sin(pi/6), cos(pi/6), and tan(pi/6).	**sin(pi/6) = 1/2, cos(pi/6) = sqrt(3)/2, tan(pi/6) = sqrt(3)/3**
2	Use triangle GHI to find sin(pi/3), cos(pi/3), and tan(pi/3).	**sin(pi/3) = sqrt(3)/2, cos(pi/3) = 1/2, tan(pi/3) = sqrt(3)**
3	Suppose sin(x) = -3/5 and cos(x) = 4/5. Find sin(pi + x), cos(pi + x), and tan(pi + x).	**sin(pi + x) = 3/5, cos(pi + x) = -4/5, tan(pi + x) = -3/4**
4	Suppose sin(x) = 5/13 and cos(x) = -12/13. Find sin(2pi - x), cos(2pi - x), and tan(2pi - x).	**sin(2pi - x) = -5/13, cos(2pi - x) = -12/13, tan(2pi - x) = 5/12**
5	At angle x, cos(x) = 7/25 and sin(x) = 24/25. Find cos(pi + x) and sin(pi + x).	**cos(pi + x) = -7/25, sin(pi + x) = -24/25**
6	At angle x, sin(x) = -8/17 and cos(x) = -15/17. Find tan(2pi - x).	**-8/15**

PART 2 · APPLY IT

1. **6sqrt(3) feet** For angle B, tan(pi/6) = AC/AB = 1/sqrt(3). Thus AC = 18/sqrt(3) = 6sqrt(3) feet.
2. **At pi + x: (15/17, -8/17). At 2pi - x: (-15/17, -8/17).** Adding pi changes the signs of both coordinates. Replacing x with 2pi - x keeps the cosine coordinate and changes the sign of the sine coordinate.

CHALLENGE

tan(pi + x) = (-sin(x))/(-cos(x)) = tan(x). Also, tan(2pi - x) = (-sin(x))/cos(x) = -tan(x), so tan(pi + x) + tan(2pi - x) = 0.

Accept equivalent exact forms, including 1/sqrt(2) for sqrt(2)/2 and 1/sqrt(3) for sqrt(3)/3. For the challenge, students must show that the two tangent expressions are opposites.