Circle Pattern Power

Use symmetry and repeating turns to evaluate trig values.

HIGH SCHOOL · HSF-TF.A.4

CCSS.Math.Content.HSF-TF.A.4 "(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions."

NAME ____________________ DATE ____________

UNIT CIRCLE PATTERNS

On the unit circle, changing x to -x reflects the point across the x-axis. This makes sine and tangent odd, sin(-x) = -sin(x) and tan(-x) = -tan(x), while cosine is even, cos(-x) = cos(x). One full turn gives sin(x + 2pi) = sin(x) and cos(x + 2pi) = cos(x); tangent repeats after pi, tan(x + pi) = tan(x), where tangent is defined.

WORKED EXAMPLE

Evaluate cos(-5pi/4).

1. Use even symmetry: cos(-5pi/4) = cos(5pi/4).
2. On the unit circle, the x-coordinate at 5pi/4 is -sqrt(2)/2.

Answer: cos(-5pi/4) = -sqrt(2)/2

PRACTICE

6 problems

1. Evaluate sin(-pi/3).

   ANSWER ____________________

2. Evaluate cos(-2pi/3).

   ANSWER ____________________

3. Evaluate tan(-pi/4).

   ANSWER ____________________

4. Evaluate sin(13pi/6).

   ANSWER ____________________

5. Evaluate cos(11pi/3).

   ANSWER ____________________

6. Evaluate tan(7pi/4).

   ANSWER ____________________

APPLY IT Show your work

1 A student music producer models a sound wave with A(t) = sin(pi t/6), where t is time in seconds measured from the beat drop. Find A(-3) and A(9).

WORK SPACE

ANSWER ____________________

2 In a skateboard video game's physics model, a rider's horizontal position is p(theta) = 4cos(theta) meters, where theta is measured in radians from the starting direction. Find p(-2pi/3) and p(2pi/3), then state why the positions match.

WORK SPACE

ANSWER ____________________

CHALLENGE One step up

CHALLENGE

Evaluate sin(-17pi/6) + cos(-13pi/3). In one sentence, explain which symmetry and periodicity rules you used.

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ANSWER KEY

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Evaluate sin(-pi/3).	**-sqrt(3)/2**
2	Evaluate cos(-2pi/3).	**-1/2**
3	Evaluate tan(-pi/4).	**-1**
4	Evaluate sin(13pi/6).	**1/2**
5	Evaluate cos(11pi/3).	**1/2**
6	Evaluate tan(7pi/4).	**-1**

PART 2 · APPLY IT

1. **A(-3) = -1 and A(9) = -1.** Substitute each time value: the angles are -pi/2 and 3pi/2, and both have sine value -1. The times are 12 seconds apart, which is one full period of the model.
2. **p(-2pi/3) = -2 m and p(2pi/3) = -2 m; cosine is even.** Because cos(-theta) = cos(theta), both inputs have the same cosine value. Since cos(2pi/3) = -1/2, multiplying by 4 gives -2 meters.

CHALLENGE

The value is 0; sine odd symmetry and cosine even symmetry, along with 2pi periodicity, reduce the terms to -1/2 and 1/2.

Accept algebraically equivalent exact forms. For the challenge, accept any one-sentence explanation that correctly uses symmetry and periodicity to obtain the two terms.