

Around the Circle

Use the unit circle to evaluate trig functions at any angle.

HIGH SCHOOL • HSF-TF.A

CCSS.Math.Content.HSF-TF.A "Extend the domain of trigonometric functions using the unit circle"

NAME _____

DATE _____

ANGLES BEYOND ONE TURN

On the unit circle, $\cos(\theta)$ is the x-coordinate and $\sin(\theta)$ is the y-coordinate. You can add or subtract 2π to find a coterminal angle between 0 and 2π . Then use $\tan(\theta) = \sin(\theta)/\cos(\theta)$, and use reciprocal functions when their denominators are not zero.

WORKED EXAMPLE

Find $\cos(11\pi/6)$.

1. Subtract one full turn: $11\pi/6 - 2\pi = 11\pi/6 - 12\pi/6 = -\pi/6$.
2. The unit-circle point at $-\pi/6$ is $(\sqrt{3}/2, -1/2)$.
3. Cosine is the x-coordinate.

Answer: $\cos(11\pi/6) = \sqrt{3}/2$

PRACTICE 6 problems

1 Find $\sin(-\pi/2)$.

ANSWER _____

2 Find $\cos(5\pi/3)$.

ANSWER _____

3 Find $\tan(4\pi/3)$.

ANSWER _____

4 Find $\sec(3\pi/4)$.

ANSWER _____

5 Find $\csc(-3\pi/2)$.

ANSWER _____

6 Find $\sin(9\pi/4)$.

ANSWER _____

APPLY IT Show your work

- 1** A music visualizer places a moving light on a unit circle. The light starts at $(1, 0)$ and rotates clockwise $5\pi/2$ radians. What are the light's final coordinates?

WORK SPACE

ANSWER _____

- 2** A game designer uses $x = \cos(\theta)$ and $y = \sin(\theta)$ to place a character on a circular path. If $\theta = 15\pi/4$, what are the character's x- and y-coordinates?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Find all angles θ in the interval $[-2\pi, 2\pi]$ for which $\sin(\theta) = -\sqrt{3}/2$. Explain how the unit circle gives each angle.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find $\sin(-\pi/2)$.	-1
2	Find $\cos(5\pi/3)$.	1/2
3	Find $\tan(4\pi/3)$.	$\sqrt{3}$
4	Find $\sec(3\pi/4)$.	$-\sqrt{2}$
5	Find $\csc(-3\pi/2)$.	1
6	Find $\sin(9\pi/4)$.	$\sqrt{2}/2$

PART 2 · APPLY IT

1. **(0, -1)** Clockwise rotation is negative, so the angle is $-5\pi/2$. This is coterminal with $-\pi/2$, whose unit-circle point is (0, -1).
2. **($\sqrt{2}/2$, $-\sqrt{2}/2$)** Subtract 2π twice: $15\pi/4 - 16\pi/4 = -\pi/4$. The unit-circle point at $-\pi/4$ is $(\sqrt{2}/2, -\sqrt{2}/2)$.

CHALLENGE

$\theta = -2\pi/3, -\pi/3, 4\pi/3, 5\pi/3$

Accept equivalent coterminal-angle reasoning and equivalent exact forms, such as $\sqrt{3}/3$ for $\tan(\pi/6)$. For the challenge, students should identify the two reference-angle locations with negative y-coordinates and list all values in the stated interval.