Inverse Angle Lab

Use restricted trig domains to match each output with one input.

HIGH SCHOOL · HSF-TF.B.6

CCSS.Math.Content.HSF-TF.B.6 "(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed."

NAME ____________________ DATE ____________

ONE DIRECTION, ONE INVERSE

An inverse function needs each output to come from only one input. Restrict a trigonometric function to an interval where it moves in one direction only, always increasing or always decreasing. On that interval, you can reverse the input-output relationship to find inverse values.

WORKED EXAMPLE

Let f(x)=cos x with 0 <= x <= pi.
Find f^-1(-1/2).

1. On 0 <= x <= pi, cos x is always decreasing, so f has an inverse.
2. f^-1(-1/2) is the x-value for which cos x=-1/2.
3. cos(2pi/3)=-1/2, and 2pi/3 is in the restricted domain.

Answer: f^-1(-1/2)=2pi/3

PRACTICE 6 problems

1 Let f(x)=sin x with -pi/2 <= x <= pi/2. Does f have an inverse? State the domain and range of f^-1.

ANSWER ____________________

2 Let g(x)=cos x with 0 <= x <= pi. Find g^-1(-sqrt(3)/2).

ANSWER ____________________

3 Let h(x)=tan x with -pi/2 < x < pi/2. Find h^-1(-sqrt(3)).

ANSWER ____________________

4 Which restriction lets sin x have an inverse: [0, pi], [-pi/2, pi/2], or [pi/4, 5pi/4]? Explain briefly.

ANSWER ____________________

5 Let p(x)=cos x with -pi <= x <= 0. Find p^-1(0).

ANSWER ____________________

6 Let q(x)=sin x with pi/2 <= x <= 3pi/2. Find q^-1(-1).

ANSWER ____________________

APPLY IT Show your work

1 During one rising part of an audio visualizer, the screen height is modeled by h=sin t, where -pi/2 <= t <= pi/2. The screen shows h=-sqrt(2)/2. What is t? Why can the app use an inverse on this interval?

WORK SPACE

ANSWER ____________________

2 In a video game, a lighting effect is modeled by b=cos x, where 0 <= x <= pi. When b=-sqrt(2)/2, what is x? Explain why this restriction gives one angle for the brightness value.

WORK SPACE

ANSWER ____________________

CHALLENGE One step up

CHALLENGE

A student restricts r(x)=cos x to pi/6 <= x <= 11pi/6 and says r has an inverse because the interval is limited. Is the student correct? Use two different inputs from the interval to explain your reasoning.

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Inverse Angle Lab

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Let f(x)=sin x with -pi/2 <= x <= pi/2. Does f have an inverse? State the domain and range of f^-1.	**Yes. The domain of f^-1 is [-1, 1], and the range of f^-1 is [-pi/2, pi/2].**
2	Let g(x)=cos x with 0 <= x <= pi. Find g^-1(-sqrt(3)/2).	**g^-1(-sqrt(3)/2)=5pi/6**
3	Let h(x)=tan x with -pi/2 < x < pi/2. Find h^-1(-sqrt(3)).	**h^-1(-sqrt(3))=-pi/3**
4	Which restriction lets sin x have an inverse: [0, pi], [-pi/2, pi/2], or [pi/4, 5pi/4]? Explain briefly.	**[-pi/2, pi/2], because sin x is always increasing on this interval.**
5	Let p(x)=cos x with -pi <= x <= 0. Find p^-1(0).	**p^-1(0)=-pi/2**
6	Let q(x)=sin x with pi/2 <= x <= 3pi/2. Find q^-1(-1).	**q^-1(-1)=3pi/2**

PART 2 · APPLY IT

1. **t=-pi/4. The app can use an inverse because sin t is always increasing on -pi/2 <= t <= pi/2.** Solve sin t=-sqrt(2)/2 within the restricted interval. The restriction makes sine one-to-one because it is increasing throughout the interval.
2. **x=3pi/4. Cosine is always decreasing on 0 <= x <= pi, so each brightness value comes from one angle.** Solve cos x=-sqrt(2)/2 on the interval from 0 to pi. Cosine decreases on that interval, so the inverse is defined there.

CHALLENGE

No. cos(pi/3)=1/2 and cos(5pi/3)=1/2, but pi/3 and 5pi/3 are different inputs in the restricted interval. Since one output has two inputs, r is not one-to-one and has no inverse on this interval.

Accept equivalent interval notation and correct monotonicity explanations. Require exact radian values that lie in the stated restricted domain.