

Signs and Squares

Use the unit circle identity to find missing trig values.

HIGH SCHOOL • HSF-TF.C.8

CCSS.Math.Content.HSF-TF.C.8 "Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle."

NAME _____

DATE _____

THE UNIT CIRCLE RULE

The unit circle has equation $x^2 + y^2 = 1$. Since $x = \cos(\theta)$ and $y = \sin(\theta)$, $\sin^2(\theta) + \cos^2(\theta) = 1$; subtract the known square, find the positive square root for the magnitude, and use the quadrant to choose the sign.

WORKED EXAMPLE

Given $\sin(\theta) = 11/61$ and θ is in Quadrant II, find $\cos(\theta)$ and $\tan(\theta)$.

- $\sin^2(\theta) + \cos^2(\theta) = 1$
- $\cos^2(\theta) = 1 - (11/61)^2 = 3600/3721$
- $\cos(\theta) = -60/61$ because cosine is negative in Quadrant II.
- $\tan(\theta) = \sin(\theta)/\cos(\theta) = (11/61)/(-60/61) = -11/60$

Answer: $\cos(\theta) = -60/61$, $\tan(\theta) = -11/60$

PRACTICE 6 problems

- 1** If $\cos(\theta) = -12/13$ and θ is in Quadrant III, find $\sin(\theta)$ and $\tan(\theta)$.

ANSWER _____

- 2** If $\tan(\theta) = -3/4$ and θ is in Quadrant II, find $\sin(\theta)$ and $\cos(\theta)$.

ANSWER _____

- 3** If $\sin(\theta) = 8/17$ and θ is in Quadrant I, find $\cos(\theta)$ and $\tan(\theta)$.

ANSWER _____

- 4** If $\tan(\theta) = -5/12$ and θ is in Quadrant IV, find $\sin(\theta)$ and $\cos(\theta)$.

ANSWER _____

- 5** If $\cos(\theta) = 7/25$ and θ is in Quadrant IV, find $\sin(\theta)$ and $\tan(\theta)$.

ANSWER _____

- 6** If $\sin(\theta) = -9/41$ and θ is in Quadrant III, find $\cos(\theta)$ and $\tan(\theta)$.

ANSWER _____

APPLY IT Show your work

- 1** At a robotics competition, a team models the direction of a camera arm with an angle θ on the unit circle. The arm has $\cos(\theta) = -8/17$ and is in Quadrant III. Find $\sin(\theta)$ and $\tan(\theta)$.

WORK SPACE

ANSWER _____

- 2** A soccer video game uses an angle θ to show the direction of a kicked ball. The ball's direction is in Quadrant IV and $\tan(\theta) = -7/24$. Find $\sin(\theta)$ and $\cos(\theta)$.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Given $\tan(\theta) = -20/21$ and θ is in Quadrant II, find $\sin(\theta)$ and $\cos(\theta)$. Then verify that your values satisfy $\sin^2(\theta) + \cos^2(\theta) = 1$.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	If $\cos(\theta) = -12/13$ and θ is in Quadrant III, find $\sin(\theta)$ and $\tan(\theta)$.	$\sin(\theta) = -5/13, \tan(\theta) = 5/12$
2	If $\tan(\theta) = -3/4$ and θ is in Quadrant II, find $\sin(\theta)$ and $\cos(\theta)$.	$\sin(\theta) = 3/5, \cos(\theta) = -4/5$
3	If $\sin(\theta) = 8/17$ and θ is in Quadrant I, find $\cos(\theta)$ and $\tan(\theta)$.	$\cos(\theta) = 15/17, \tan(\theta) = 8/15$
4	If $\tan(\theta) = -5/12$ and θ is in Quadrant IV, find $\sin(\theta)$ and $\cos(\theta)$.	$\sin(\theta) = -5/13, \cos(\theta) = 12/13$
5	If $\cos(\theta) = 7/25$ and θ is in Quadrant IV, find $\sin(\theta)$ and $\tan(\theta)$.	$\sin(\theta) = -24/25, \tan(\theta) = -24/7$
6	If $\sin(\theta) = -9/41$ and θ is in Quadrant III, find $\cos(\theta)$ and $\tan(\theta)$.	$\cos(\theta) = -40/41, \tan(\theta) = 9/40$

PART 2 · APPLY IT

- $\sin(\theta) = -15/17, \tan(\theta) = 15/8$** Use $\sin^2(\theta) = 1 - (-8/17)^2$ to get a sine magnitude of 15/17. Quadrant III makes both sine and cosine negative, so tangent is positive.
- $\sin(\theta) = -7/25, \cos(\theta) = 24/25$** Use a reference triangle with opposite 7, adjacent 24, and hypotenuse 25. In Quadrant IV, sine is negative and cosine is positive.

CHALLENGE

$\sin(\theta) = 20/29$ and $\cos(\theta) = -21/29$. Verification: $(20/29)^2 + (-21/29)^2 = 400/841 + 441/841 = 1$.

Accept equivalent simplified fraction forms. For the challenge, accept any correct verification showing the two squared fractions add to 1.