Angle Formula Lab

Build exact values from angle sums and differences.

HIGH SCHOOL · HSF-TF.C.9

CCSS.Math.Content.HSF-TF.C.9 "(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems."

NAME ______________________ DATE ____________

COMBINE KNOWN ANGLES

Use sin(a+b)=sin a cos b+cos a sin b, sin(a-b)=sin a cos b-cos a sin b, cos(a+b)=cos a cos b-sin a sin b, and cos(a-b)=cos a cos b+sin a sin b. Also, tan(a+b)=(tan a+tan b)/(1-tan a tan b) and tan(a-b)=(tan a-tan b)/(1+tan a tan b), when each expression is defined.

WORKED EXAMPLE

Given sin p=8/17, cos p=15/17, sin q=7/25, and cos q=24/25, find cos(p+q).

1. Use cos(p+q)=cos p cos q-sin p sin q.
2. Substitute: cos(p+q)=(15/17)(24/25)-(8/17)(7/25).
3. Multiply: cos(p+q)=360/425-56/425.
4. Subtract: cos(p+q)=304/425.

Answer: 304/425

PRACTICE 6 problems

1 Find the exact value of sin(165 degrees).

ANSWER ____________

2 Find the exact value of cos(105 degrees).

ANSWER ____________

3 Find the exact value of tan(75 degrees).

ANSWER ____________

4 Angles a and b are acute. If sin a=3/5, cos a=4/5, sin b=5/13, and cos b=12/13, find cos(a+b).

ANSWER ____________

5 Angle x is in Quadrant II and angle y is acute. If sin x=20/29, cos x=-21/29, sin y=9/41, and cos y=40/41, find sin(x-y).

ANSWER ____________

6 Use addition and subtraction formulas to simplify sin(x+y)+sin(x-y).

ANSWER ____________

APPLY IT
Show your work

1 In a racing game, a character is set to move 12 virtual units at an angle of 45 degrees above the positive x-axis. A power-up turns the character 90 degrees counterclockwise before the move. What is the exact horizontal component of the move?

WORK SPACE

ANSWER ______________________

2 A student running stage lighting aims a spotlight 10 meters in a direction of 150 degrees from the positive horizontal. A lighting preset shifts the beam 45 degrees counterclockwise. What is the exact horizontal component of the beam's position?

WORK SPACE

ANSWER ______________________

CHALLENGE
One step up

CHALLENGE

Starting with tan(a-b)=sin(a-b)/cos(a-b), prove that tan(a-b)=(tan a-tan b)/(1+tan a tan b). State the restrictions needed for this formula.

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the exact value of sin(165 degrees).	**(sqrt(6)-sqrt(2))/4**
2	Find the exact value of cos(105 degrees).	**(sqrt(2)-sqrt(6))/4**
3	Find the exact value of tan(75 degrees).	**2+sqrt(3)**
4	Angles a and b are acute. If sin a=3/5, cos a=4/5, sin b=5/13, and cos b=12/13, find cos(a+b).	**33/65**
5	Angle x is in Quadrant II and angle y is acute. If sin x=20/29, cos x=-21/29, sin y=9/41, and cos y=40/41, find sin(x-y).	**989/1189**
6	Use addition and subtraction formulas to simplify sin(x+y)+sin(x-y).	**2 sin x cos y**

PART 2 · APPLY IT

1. **-6sqrt(2) virtual units** The new direction is 45 degrees+90 degrees=135 degrees. The horizontal component is 12 cos(135 degrees)=12(-sqrt(2)/2)=-6sqrt(2).
2. **-(5/2)(sqrt(6)+sqrt(2)) meters** The final direction is 150 degrees+45 degrees=195 degrees. The horizontal component is 10 cos(195 degrees)=-(5/2)(sqrt(6)+sqrt(2)).

CHALLENGE

tan(a-b)=[sin a cos b-cos a sin b]/[cos a cos b+sin a sin b]. Divide the numerator and denominator by cos a cos b to get (tan a-tan b)/(1+tan a tan b). Require cos a!=0, cos b!=0, and 1+tan a tan b!=0.

Accept algebraically equivalent exact forms, including rationalized or unrationalized radical forms. For the challenge, students should show substitution of the sine and cosine difference formulas and identify restrictions that keep all denominators nonzero.