LESSON MAKER PRO

Identity Power Moves

Use trig relationships to simplify expressions and find exact values.

HIGH SCHOOL · HSF-TF.C

CCSS.Math.Content.HSF-TF.C "Prove and apply trigonometric identities"

NAME _____ DATE _____

CONNECT THE RATIOS

The Pythagorean identities $\sin^2 \theta + \cos^2 \theta = 1$ and $1 + \tan^2 \theta = \sec^2 \theta$ connect trigonometric functions. The quotient identity $\tan \theta = \sin \theta / \cos \theta$ lets you replace a quotient with one trig function, then simplify one step at a time.

WORKED EXAMPLE

Simplify $\sin u / \cos u$.

1. Start with $\sin u / \cos u$.
2. Use the quotient identity $\tan u = \sin u / \cos u$.
3. Replace the quotient with $\tan u$.

Answer: tan u

PRACTICE 6 problems

1 Simplify $(\sin^2 a + \cos^2 a)\tan a$.

ANSWER _____

2 Simplify $(1 - \cos^2 b) / \sin b$.

ANSWER _____

3 Simplify $(\tan c)(\cos c)$.

ANSWER _____

4 Simplify $(\sin d)(\csc d) + (\cos d)(\sec d)$.

ANSWER _____

5 If $\sin e = 5/13$ and e is acute, find $\cos e$.

ANSWER _____

6 If $\tan f = 7/24$ and f is acute, find $\sec f$.

ANSWER _____

APPLY IT Show your work

1 For a stage-lighting design, a beam makes an acute angle θ with the floor. The beam's rise-to-run ratio is tan θ = 8/15. Find the exact value of sin θ.

WORK SPACE

ANSWER _____

2 A robotics team programs an arm to move at an acute angle φ. Its horizontal component ratio is cos φ = 20/29. Find the exact value of tan φ.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Prove that (1 - cos x) / sin x = sin x / (1 + cos x) for every x where both sides are defined. Show algebra steps and name the identity you use.

WORK SPACE AND ANSWER

ANSWER KEY

Identity Power Moves

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Simplify $(\sin^2 a + \cos^2 a)\tan a$.	**tan a**
2	Simplify $(1 - \cos^2 b) / \sin b$.	**sin b**
3	Simplify $(\tan c)(\cos c)$.	**sin c**
4	Simplify $(\sin d)(\csc d) + (\cos d)(\sec d)$.	**2**
5	If $\sin e = 5/13$ and e is acute, find $\cos e$.	**12/13**
6	If $\tan f = 7/24$ and f is acute, find $\sec f$.	**25/24**

PART 2 · APPLY IT

1. **8/17** Use opposite = 8 and adjacent = 15. The hypotenuse is 17, so $\sin \theta = 8/17$.
2. **21/20** Use $\sin^2 \varphi + \cos^2 \varphi = 1$ to find $\sin \varphi = 21/29$. Then $\tan \varphi = (21/29) / (20/29) = 21/20$.

CHALLENGE

Multiply the left side by $(1 + \cos x) / (1 + \cos x)$: $(1 - \cos x)(1 + \cos x) / [\sin x(1 + \cos x)] = (1 - \cos^2 x) / [\sin x(1 + \cos x)] = \sin^2 x / [\sin x(1 + \cos x)] = \sin x / (1 + \cos x)$. This uses $\sin^2 x + \cos^2 x = 1$.

Accept equivalent forms that simplify to the listed answers. For identity simplification, assume all displayed expressions are defined.