

# Circle Angle Logic

Use opposite angles to solve cyclic quadrilaterals.

**HIGH SCHOOL • HSG-C.A.3**

**CCSS.Math.Content.HSG-C.A.3** "Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle."

NAME \_\_\_\_\_

DATE \_\_\_\_\_

## OPPOSITE ANGLES

A quadrilateral is cyclic when all four of its vertices lie on one circle. In a cyclic quadrilateral, each pair of opposite interior angles adds to 180 degrees. Set opposite-angle expressions equal to a total of 180, then solve.

## WORKED EXAMPLE

Cyclic quadrilateral WXYZ has  $m \angle W = 68$  degrees. Find  $m \angle Y$ .

- Angles W and Y are opposite angles.
- $m \angle W + m \angle Y = 180$  degrees.
- $68 + m \angle Y = 180$ , so  $m \angle Y = 112$  degrees.

**Answer:  $m \angle Y = 112$  degrees**

## PRACTICE 6 problems

- 1** Cyclic quadrilateral ABCD has  $m \angle A = 73$  degrees. Find  $m \angle C$ .

**ANSWER** \_\_\_\_\_

- 2** In cyclic quadrilateral EFGH,  $m \angle F = 4x + 8$  and  $m \angle H = 7x - 4$ . Find  $x$  and both angle measures.

**ANSWER** \_\_\_\_\_

- 3** In cyclic quadrilateral JKLM,  $m \angle J = x + 21$  and  $m \angle L = 3x - 1$ . Find  $x$  and  $m \angle J$ .

**ANSWER** \_\_\_\_\_

- 4** Cyclic quadrilateral PQRS has  $m \angle P = 48$  degrees,  $m \angle Q = 95$  degrees, and  $m \angle R = 132$  degrees. Find  $m \angle S$ .

**ANSWER** \_\_\_\_\_

- 5** An exterior angle at A of cyclic quadrilateral ABCD measures 126 degrees. Find  $m \angle C$ .

**ANSWER** \_\_\_\_\_

- 6** A convex quadrilateral has angle measures 84 degrees, 101 degrees, 96 degrees, and 79 degrees in order around the quadrilateral. Can it be inscribed in a circle? Explain.

**ANSWER** \_\_\_\_\_

## APPLY IT Show your work

- 1** In a game design class, four checkpoints P, Q, R, and S lie on the edge of a circular virtual arena. The checkpoints form cyclic quadrilateral PQRS. Angle P measures  $7x + 3$  degrees, and opposite angle R measures  $5x + 21$  degrees. Find  $x$ ,  $m\angle P$ , and  $m\angle R$ .

WORK SPACE

ANSWER \_\_\_\_\_

- 2** A robotics team places sensors A, B, C, and D around the rim of a circular practice arena. The sensors form cyclic quadrilateral ABCD. Angle B measures  $3x + 12$  degrees, and opposite angle D measures  $5x + 8$  degrees. Find  $x$  and the two angle measures.

WORK SPACE

ANSWER \_\_\_\_\_

## CHALLENGE One step up

### CHALLENGE

Quadrilateral ABCD is inscribed in a circle, in that order. The arc measures are  $\text{arc AB} = 74$  degrees,  $\text{arc BC} = 96$  degrees,  $\text{arc CD} = 110$  degrees, and  $\text{arc DA} = 80$  degrees. Use the inscribed-angle theorem to show that  $m\angle A + m\angle C = 180$  degrees.

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## PART 1 • PRACTICE

| # | PROBLEM                                                                                                                                                                    | ANSWER                                                                                                                                               |
|---|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1 | Cyclic quadrilateral ABCD has $m\angle A = 73$ degrees. Find $m\angle C$ .                                                                                                 | <b><math>m\angle C = 107</math> degrees</b>                                                                                                          |
| 2 | In cyclic quadrilateral EFGH, $m\angle F = 4x + 8$ and $m\angle H = 7x - 4$ . Find $x$ and both angle measures.                                                            | <b><math>x = 16</math>; <math>m\angle F = 72</math> degrees; <math>m\angle H = 108</math> degrees</b>                                                |
| 3 | In cyclic quadrilateral JKLM, $m\angle J = x + 21$ and $m\angle L = 3x - 1$ . Find $x$ and $m\angle J$ .                                                                   | <b><math>x = 40</math>; <math>m\angle J = 61</math> degrees</b>                                                                                      |
| 4 | Cyclic quadrilateral PQRS has $m\angle P = 48$ degrees, $m\angle Q = 95$ degrees, and $m\angle R = 132$ degrees. Find $m\angle S$ .                                        | <b><math>m\angle S = 85</math> degrees</b>                                                                                                           |
| 5 | An exterior angle at A of cyclic quadrilateral ABCD measures 126 degrees. Find $m\angle C$ .                                                                               | <b><math>m\angle C = 126</math> degrees</b>                                                                                                          |
| 6 | A convex quadrilateral has angle measures 84 degrees, 101 degrees, 96 degrees, and 79 degrees in order around the quadrilateral. Can it be inscribed in a circle? Explain. | <b>Yes. The opposite angle pairs are <math>84 + 96 = 180</math> degrees and <math>101 + 79 = 180</math> degrees, so the quadrilateral is cyclic.</b> |

## PART 2 • APPLY IT

- $x = 13$ ;  $m\angle P = 94$  degrees;  $m\angle R = 86$  degrees** Opposite angles in a cyclic quadrilateral are supplementary, so  $(7x + 3) + (5x + 21) = 180$ . Solving gives  $x = 13$ , then substitute to find the two angles.
- $x = 20$ ;  $m\angle B = 72$  degrees;  $m\angle D = 108$  degrees** Set the opposite angles equal to 180 degrees:  $(3x + 12) + (5x + 8) = 180$ . Solve for  $x$ , then substitute into each expression.

## CHALLENGE

**Angle A intercepts arc BCD, so  $m\angle A = \frac{1}{2}(96 + 110) = 103$  degrees. Angle C intercepts arc DAB, so  $m\angle C = \frac{1}{2}(80 + 74) = 77$  degrees. Therefore,  $m\angle A + m\angle C = 103 + 77 = 180$  degrees.**

Accept equivalent equations and angle labels. For the explanation item, students should state that both pairs of opposite angles are supplementary.