

Touch Point Paths

Construct tangent lines from an outside point.

HIGH SCHOOL • HSG-C.A.4

CCSS.Math.Content.HSG-C.A.4 "(+) Construct a tangent line from a point outside a given circle to the circle."

NAME _____

DATE _____

BUILD A TANGENT LINE

A tangent line touches a circle at exactly one point, and the radius to that touch point is perpendicular to the tangent line. In a coordinate construction, make a line through the outside point and choose its slope so the perpendicular distance from the center to the line equals the radius.

WORKED EXAMPLE

Construct the two tangent lines from $P(25, 0)$ to the circle $x^2 + y^2 = 49$.

1. A line through P has the form $y = m(x - 25)$.
2. Its distance from the center must be 7, so $25|m| / \sqrt{m^2 + 1} = 7$.
3. Squaring gives $625m^2 = 49(m^2 + 1)$, so $m = 7/24$ or $m = -7/24$.
4. Substitute each slope into $y = m(x - 25)$.

Answer: $y = 7/24(x - 25)$ and $y = -7/24(x - 25)$

PRACTICE 6 problems

- 1** Construct both tangent lines from $P(5, 0)$ to the circle $x^2 + y^2 = 9$. Write each line equation.

ANSWER _____

- 2** Construct both tangent lines from $P(0, 5)$ to the circle $x^2 + y^2 = 16$. Write each line equation.

ANSWER _____

- 3** Construct both tangent lines from $P(6, -1)$ to the circle centered at $(1, -1)$ with radius 3. Write each line equation.

ANSWER _____

- 4** Construct both tangent lines from $P(1, 3)$ to the circle $(x - 1)^2 + (y + 2)^2 = 16$. Write each line equation.

ANSWER _____

- 5** Construct both tangent lines from $P(11, 3)$ to the circle centered at $(-2, 3)$ with radius 5. Write each line equation.

ANSWER _____

- 6** Construct both tangent lines from $P(3, 11)$ to the circle centered at $(3, 1)$ with radius 6. Write each line equation.

ANSWER _____

APPLY IT Show your work

- 1** In a game-design class, a circular safety zone has center $(1, 1)$ and radius 8 map units. A drone launches from $P(18, 1)$. Construct the equations of the two straight flight paths that touch, but do not enter, the safety zone.

WORK SPACE

ANSWER _____

- 2** A robotics club maps a circular charging area with center $(4, 2)$ and radius 5 meters. A robot begins at $P(4, 15)$. Construct the equations of the two straight paths that just touch the charging area.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Construct both tangent lines from $P(13, 6)$ to the circle centered at $(1, 1)$ with radius 5. Then explain how you know each line is tangent.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Construct both tangent lines from P(5, 0) to the circle $x^2 + y^2 = 9$. Write each line equation.	$3x + 4y = 15$ and $3x - 4y = 15$
2	Construct both tangent lines from P(0, 5) to the circle $x^2 + y^2 = 16$. Write each line equation.	$y = 5 + 3/4x$ and $y = 5 - 3/4x$
3	Construct both tangent lines from P(6, -1) to the circle centered at (1, -1) with radius 3. Write each line equation.	$y + 1 = 3/4(x - 6)$ and $y + 1 = -3/4(x - 6)$
4	Construct both tangent lines from P(1, 3) to the circle $(x - 1)^2 + (y + 2)^2 = 16$. Write each line equation.	$y - 3 = 3/4(x - 1)$ and $y - 3 = -3/4(x - 1)$
5	Construct both tangent lines from P(11, 3) to the circle centered at (-2, 3) with radius 5. Write each line equation.	$y - 3 = 5/12(x - 11)$ and $y - 3 = -5/12(x - 11)$
6	Construct both tangent lines from P(3, 11) to the circle centered at (3, 1) with radius 6. Write each line equation.	$y - 11 = 4/3(x - 3)$ and $y - 11 = -4/3(x - 3)$

PART 2 • APPLY IT

1. $y - 1 = 8/15(x - 18)$ and $y - 1 = -8/15(x - 18)$ The launch point is 17 units from the center. Set the distance from the center to a line through P equal to 8, which gives slopes $8/15$ and $-8/15$.
2. $y - 15 = 12/5(x - 4)$ and $y - 15 = -12/5(x - 4)$ The robot is 13 meters from the center. A line through P must stay 5 meters from the center, giving slopes $12/5$ and $-12/5$.

CHALLENGE

$y = 6$ and $120x - 119y = 846$. Both lines pass through P, and the perpendicular distance from the center (1, 1) to each line is 5, the circle's radius, so each line touches the circle at exactly one point.

Accept equivalent equations in slope-intercept, point-slope, or standard form. For each constructed line, verify that it passes through the outside point and that its distance from the circle's center equals the radius.