

# Circle Slice Lab

*Use radians to measure arcs and sectors.*

**HIGH SCHOOL • HSG-C.B.5**

**CCSS.Math.Content.HSG-C.B.5** "Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector."

NAME \_\_\_\_\_

DATE \_\_\_\_\_

## RADIANS CONNECT LENGTH AND AREA

For the same central angle in similar circles, arc length divided by radius stays constant. That constant is the angle measure in radians, so  $s = r\theta$ . A sector is the same fraction of a circle as its angle is of  $2\pi$  radians, so  $A = \frac{1}{2} r^2\theta$ .

## WORKED EXAMPLE

A sector has radius 13 cm and central angle  $\pi/7$  radians. Find its arc length and area.

1. Use  $s = r\theta$ .
2.  $s = 13(\pi/7) = 13\pi/7$  cm.
3. Use  $A = \frac{1}{2} r^2\theta$ .
4.  $A = \frac{1}{2}(13^2)(\pi/7) = 169\pi/14$  cm<sup>2</sup>.

**Answer: Arc length:  $13\pi/7$  cm; area:  $169\pi/14$  cm<sup>2</sup>**

## PRACTICE 6 problems

- 1** A circle has radius 6 cm. Find the arc length intercepted by a central angle of  $\pi/4$  radians.

**ANSWER** \_\_\_\_\_

- 2** Find the area of a sector with radius 5 m and central angle  $3\pi/5$  radians.

**ANSWER** \_\_\_\_\_

- 3** An arc is  $15\pi$  cm long in a circle with radius 20 cm. Find the central angle in radians.

**ANSWER** \_\_\_\_\_

- 4** A sector has radius 8 in. and area  $64\pi$  in<sup>2</sup>. Find its central angle in radians.

**ANSWER** \_\_\_\_\_

- 5** A sector has radius 12 ft and central angle  $5\pi/6$  radians. Find its arc length and area.

**ANSWER** \_\_\_\_\_

- 6** A sector has radius 9 cm and area  $27\pi/4$  cm<sup>2</sup>. Find its central angle in radians.

**ANSWER** \_\_\_\_\_

## APPLY IT Show your work

- 1** At a music festival, a spotlight is 25 m from the center of a circular stage. It sweeps through an angle of  $3\pi/5$  radians. What area of the stage can the spotlight cover?

WORK SPACE

ANSWER \_\_\_\_\_

- 2** In a drone racing game, a camera moves along a circular path with radius 56 ft. The camera travels an arc length of  $28\pi$  ft. Find the angle it turns in radians and the area of the sector it sweeps.

WORK SPACE

ANSWER \_\_\_\_\_

## CHALLENGE One step up

### CHALLENGE

Sector P has radius 15 cm and central angle  $2\pi/3$  radians. Sector Q has radius 10 cm and the same arc length as Sector P. Find the angle and area of Sector Q. Then explain why Sector P has the greater area even though the arc lengths are equal.

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## PART 1 • PRACTICE

| # | PROBLEM                                                                                          | ANSWER                                                                            |
|---|--------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| 1 | A circle has radius 6 cm. Find the arc length intercepted by a central angle of $\pi/4$ radians. | <b><math>3\pi/2</math> cm</b>                                                     |
| 2 | Find the area of a sector with radius 5 m and central angle $3\pi/5$ radians.                    | <b><math>15\pi/2</math> m<sup>2</sup></b>                                         |
| 3 | An arc is $15\pi$ cm long in a circle with radius 20 cm. Find the central angle in radians.      | <b><math>3\pi/4</math> radians</b>                                                |
| 4 | A sector has radius 8 in. and area $64\pi$ in <sup>2</sup> . Find its central angle in radians.  | <b><math>2\pi</math> radians</b>                                                  |
| 5 | A sector has radius 12 ft and central angle $5\pi/6$ radians. Find its arc length and area.      | <b>Arc length: <math>10\pi</math> ft; area: <math>60\pi</math> ft<sup>2</sup></b> |
| 6 | A sector has radius 9 cm and area $27\pi/4$ cm <sup>2</sup> . Find its central angle in radians. | <b><math>\pi/6</math> radians</b>                                                 |

## PART 2 • APPLY IT

1.  **$375\pi/2$  m<sup>2</sup>** Use  $A = \frac{1}{2} r^2 \theta$ .  $A = \frac{1}{2}(25^2)(3\pi/5) = 375\pi/2$  m<sup>2</sup>.

2. **Angle:  $\pi/2$  radians; area:  $784\pi$  ft<sup>2</sup>** Use  $\theta = s/r = 28\pi/56 = \pi/2$ . Then  $A = \frac{1}{2}(56^2)(\pi/2) = 784\pi$  ft<sup>2</sup>.

## CHALLENGE

**Sector P has arc length  $15(2\pi/3) = 10\pi$  cm. Sector Q has angle  $10\pi/10 = \pi$  radians and area  $\frac{1}{2}(10^2)(\pi) = 50\pi$  cm<sup>2</sup>. Sector P has area  $\frac{1}{2}(15^2)(2\pi/3) = 75\pi$  cm<sup>2</sup>, so it has the greater area because the same arc length reaches farther from the center and encloses more region.**

Accept equivalent exact forms, such as  $1.5\pi$  for  $3\pi/2$ , when units are included. Require radians for angle answers and square units for area answers.