LESSON MAKER PRO

Diagonal Detectives

Use parallelogram facts to solve for missing measures.

HIGH SCHOOL · HSG-CO.C.11

CCSS.Math.Content.HSG-CO.C.11 "Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals."

NAME _____ DATE _____

PARALLELOGRAM FACTS

In a parallelogram, opposite sides have the same length, and opposite angles have the same measure. Its diagonals bisect each other, so each diagonal is split into two equal parts. If a parallelogram has diagonals with the same length, it is a rectangle.

WORKED EXAMPLE

PQRS is a parallelogram. Diagonals PR and QS intersect at T. PT = 7x - 5 and TR = 3x + 19. Find x and PR.

1. The diagonals bisect each other, so PT = TR.
2. Set the expressions equal: 7x - 5 = 3x + 19.
3. Solve: 4x = 24, so x = 6.
4. PT = 37, so PR = 74.

Answer: x = 6; PR = 74

PRACTICE 6 problems

1 ABCD is a parallelogram. AB = 8x + 2 and CD = 10x - 20. Find x and AB.

ANSWER _____

2 ABCD is a parallelogram. m∠A = 9x + 12 and m∠C = 13x - 28. Find x and m∠A.

ANSWER _____

3 ABCD is a parallelogram. Diagonals AC and BD intersect at E. AE = 12x + 1 and EC = 8x + 33. Find x and AC.

ANSWER _____

4 WXYZ is a rectangle. WY = 14x - 16 and XZ = 10x + 20. Find x and the length of each diagonal.

ANSWER _____

5 JKLM is a parallelogram. Its diagonals have lengths JL = 86 and KM = 86. What can you conclude about JKLM?

ANSWER _____

6 A parallelogram has adjacent side lengths 12x + 1 and 8x + 9. Its perimeter is 340. Find x and both side lengths.

ANSWER _____

APPLY IT Show your work

1 A student is designing a parallelogram-shaped decal for a skate club. Its diagonals meet at M. One half of a diagonal is 13x + 2 cm, and the opposite half is 9x + 34 cm. Find x and the full length of that diagonal.

WORK SPACE

ANSWER _____

2 A gamer is comparing measurements for a rectangular monitor. Its diagonals are 16x + 11 cm and 20x - 25 cm. Find x and the length of each diagonal.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

ABCD is a parallelogram with congruent diagonals. AC = 15x + 2 and BD = 11x + 38. Find x. Then explain why ABCD must be a rectangle.

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_____

ANSWER KEY

Diagonal Detectives

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	ABCD is a parallelogram. AB = 8x + 2 and CD = 10x - 20. Find x and AB.	**x = 11; AB = 90**
2	ABCD is a parallelogram. m∠A = 9x + 12 and m∠C = 13x - 28. Find x and m∠A.	**x = 10; m∠A = 102 degrees**
3	ABCD is a parallelogram. Diagonals AC and BD intersect at E. AE = 12x + 1 and EC = 8x + 33. Find x and AC.	**x = 8; AC = 194**
4	WXYZ is a rectangle. WY = 14x - 16 and XZ = 10x + 20. Find x and the length of each diagonal.	**x = 9; each diagonal = 110**
5	JKLM is a parallelogram. Its diagonals have lengths JL = 86 and KM = 86. What can you conclude about JKLM?	**JKLM is a rectangle.**
6	A parallelogram has adjacent side lengths 12x + 1 and 8x + 9. Its perimeter is 340. Find x and both side lengths.	**x = 8; the side lengths are 97 and 73**

PART 2 · APPLY IT

1. **x = 8; the full diagonal is 212 cm** Set the two half-diagonal expressions equal because diagonals of a parallelogram bisect each other. Each half is 106 cm, so the full diagonal is 212 cm.
2. **x = 9; each diagonal is 155 cm** Set the diagonal expressions equal because diagonals of a rectangle are congruent. Substitute x = 9 into either expression to get 155 cm.

CHALLENGE

x = 9. ABCD is a rectangle because it is a parallelogram with congruent diagonals.

Accept equivalent statements such as “opposite angles are congruent” and angle measures written with or without a degree symbol. For the challenge, require both x = 9 and the congruent-diagonals justification.