

Congruence Clues

Use matching triangle parts to solve and prove.

HIGH SCHOOL • HSG-CO.C

CCSS.Math.Content.HSG-CO.C "Prove geometric theorems"

NAME _____

DATE _____

MATCH CORRESPONDING PARTS

When triangles are congruent, their matching sides and angles have equal measures. Match vertices by their order in the congruence statement, then set corresponding expressions equal. This idea is called CPCTC, corresponding parts of congruent triangles are congruent.

WORKED EXAMPLE

Given $\triangle GHI \cong \triangle KLM$, $GH = 4x + 11$ and $KL = 6x - 1$. Find x and the length of GH .

1. GH corresponds to KL , so $4x + 11 = 6x - 1$.
2. Add 1 and subtract $4x$: $12 = 2x$.
3. Divide by 2: $x = 6$.
4. Substitute: $GH = 4(6) + 11 = 35$.

Answer: $x = 6$; $GH = KL = 35$

PRACTICE 6 problems

- 1** Given $\triangle ABC \cong \triangle RST$, $AB = 7x + 8$ and $RS = 9x - 18$. Find x and the length of AB .

ANSWER _____

- 2** Given $\triangle UVW \cong \triangle PQR$, $\angle V = 7y - 13$ and $\angle Q = 3y + 19$. Find y and $m\angle V$.

ANSWER _____

- 3** Given $\triangle MNO \cong \triangle XYZ$, $MO = 5a + 9$ and $XZ = 3a + 25$. Find a and the length of MO .

ANSWER _____

- 4** Given $\triangle DEF \cong \triangle TUV$, name the angle congruent to $\angle E$.

ANSWER _____

- 5** Given $\triangle HIJ \cong \triangle LMN$, $\angle J = 9m - 16$ and $\angle N = 5m + 20$. Find m and $m\angle J$.

ANSWER _____

- 6** In isosceles $\triangle PQS$, $PQ = PS$. If $\angle Q = 5t + 7$ and $\angle S = 3t + 21$, find t and the measure of each base angle.

ANSWER _____

APPLY IT Show your work

- 1** The student council is designing congruent triangular light frames for a concert. Corresponding support bars on the frames measure $3x + 15$ cm and $5x - 21$ cm. Find x and the actual length of each support bar.

WORK SPACE

ANSWER _____

- 2** A skate park uses congruent triangular side panels for two ramps. Corresponding top angles are labeled $7n + 3$ degrees and $5n + 17$ degrees. Find n and the measure of each top angle.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

In $\triangle ABC$, $AB = AC$. Point D lies on BC , and AD bisects $\angle BAC$. Explain why $\triangle ABD \cong \triangle ACD$. Then, if $BD = 8x - 17$ and $DC = 3x + 28$, find x and the length of BD .

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Given $\triangle ABC \cong \triangle RST$, $AB = 7x + 8$ and $RS = 9x - 18$. Find x and the length of AB .	$x = 13$; $AB = RS = 99$
2	Given $\triangle UVW \cong \triangle PQR$, $\angle V = 7y - 13$ and $\angle Q = 3y + 19$. Find y and $m\angle V$.	$y = 8$; $m\angle V = m\angle Q = 43^\circ$
3	Given $\triangle MNO \cong \triangle XYZ$, $MO = 5a + 9$ and $XZ = 3a + 25$. Find a and the length of MO .	$a = 8$; $MO = XZ = 49$
4	Given $\triangle DEF \cong \triangle TUV$, name the angle congruent to $\angle E$.	$\angle U$
5	Given $\triangle HIJ \cong \triangle LMN$, $\angle J = 9m - 16$ and $\angle N = 5m + 20$. Find m and $m\angle J$.	$m = 9$; $m\angle J = m\angle N = 65^\circ$
6	In isosceles $\triangle PQS$, $PQ = PS$. If $\angle Q = 5t + 7$ and $\angle S = 3t + 21$, find t and the measure of each base angle.	$t = 7$; $m\angle Q = m\angle S = 42^\circ$

PART 2 · APPLY IT

- $x = 18$; each support bar is 69 cm long** Because the frames are congruent, the corresponding support bars are equal. Set $3x + 15$ equal to $5x - 21$, then substitute $x = 18$.
- $n = 7$; each top angle measures 52°** Corresponding angles in congruent panels have equal measures. Set $7n + 3$ equal to $5n + 17$, then substitute $n = 7$.

CHALLENGE

$AB = AC$, $AD = AD$, and $\angle BAD = \angle DAC$ because AD bisects $\angle BAC$. Therefore, $\triangle ABD \cong \triangle ACD$ by SAS , so $BD = DC$ by $CPCTC$. Set $8x - 17 = 3x + 28$ to get $x = 9$, and $BD = DC = 55$.

Accept equivalent equation-solving work and standard angle notation. For the challenge, require the equal sides, the shared side AD , the equal bisected angles, SAS congruence, and then $CPCTC$ to justify $BD = DC$.