

Build the Formula

Use cuts, layers, and matching solids to justify circle and volume formulas.

HIGH SCHOOL • HSG-GMD.A.1

CCSS.Math.Content.HSG-GMD.A.1 "Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments."

NAME _____

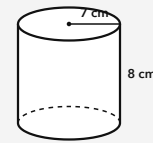
DATE _____

CUT, REARRANGE, COMPARE

The distance around a circle is $C=2\pi r$. Cut a circle into many equal sectors and alternate them to make a shape close to a rectangle with base πr and height r , so $A=\pi r^2$. A cylinder has volume base area times height, while a cone or pyramid with the same base and height has $1/3$ that volume.

WORKED EXAMPLE

Find the volume of the cylinder shown.

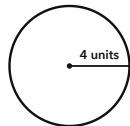


1. Base area = $\pi(7)^2 = 49\pi \text{ cm}^2$.
2. Volume = base area $\times$ height.
3. $V = 49\pi(8) = 392\pi \text{ cm}^3$.
- 4.

Answer: $392\pi \text{ cm}^3$

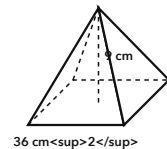
PRACTICE 4 problems

- 1** The circle shown is cut into many equal sectors and alternated. Find its area. Then name the base and height of the near-rectangle.



ANSWER _____

- 2** The figure labels a pyramid's base area and perpendicular height. Find its volume.



ANSWER _____

- 3** A cone has radius 3 cm and height 10 cm. Find its volume. State how it compares with a cylinder of the same radius and height.

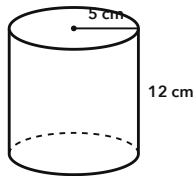
ANSWER _____

- 4** A circular track has diameter 12 m. Find its circumference and area.

ANSWER _____

APPLY IT Show your work

- 1** A robotics team fills the cylindrical ballast tube shown with sand. Find the tube's volume.



ANSWER _____

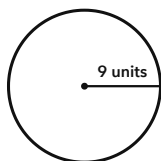
- 2** A theater club makes a cone-shaped spotlight cover with radius 6 cm and perpendicular height 15 cm. Find its volume.

WORK SPACE

ANSWER _____

CHALLENGE One step up

The circle shown has radius 9 units. It is cut into many congruent sectors and alternated. Explain why the near-rectangle has base πr and height r . Then find the circle's area.



WORK SPACE AND ANSWER

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	The circle shown is cut into many equal sectors and alternated. Find its area. Then name the base and height of the near-rectangle.	16n square units; base 4n units and height 4 units
2	The figure labels a pyramid's base area and perpendicular height. Find its volume.	108 cm³
3	A cone has radius 3 cm and height 10 cm. Find its volume. State how it compares with a cylinder of the same radius and height.	30n cm³; it is 1/3 of the cylinder's volume
4	A circular track has diameter 12 m. Find its circumference and area.	Circumference = 12n m; area = 36n m²

PART 2 • APPLY IT

1. **300n cm³** Use $V = \pi r^2 h$. Substitute 5 for r and 12 for h.
2. **180n cm³** Use $V = (1/3)\pi r^2 h$. Substitute 6 for r and 15 for h.

CHALLENGE

The two sets of curved edges make a base equal to half the circumference, πr , and the radii make the height r. $A = \pi r \cdot r = 81\pi$ square units.

Accept equivalent unit wording and correctly simplified or unsimplified exact π expressions. For the challenge, accept any informal explanation that identifies half the circumference as the base and the radius as the height.