

Slice Match Volume

Use matching cross sections to find volume.

HIGH SCHOOL • HSG-GMD.A.2

CCSS.Math.Content.HSG-GMD.A.2 "(+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures."

NAME _____

DATE _____

MATCH THE SLICES

If two solids have the same height and equal-area cross sections at every matching height, they have equal volumes. For a sphere, Cavalieri's principle lets you find its volume by comparing it with a solid made from familiar pieces.

WORKED EXAMPLE

Find the volume of a sphere with radius 6 units.

1. Use $V = \frac{4}{3} \pi r^3$.
2. Substitute: $V = \frac{4}{3} \pi (6^3)$.
3. Simplify: $V = 288\pi$.

Answer: 288π cubic units

PRACTICE 5 problems

- 1** Find the volume of a sphere with radius 3 units.

ANSWER _____

- 2** Find the volume of a sphere with diameter 8 units.

ANSWER _____

- 3** A comparison solid is a cylinder of radius 2 and height 4 with two cones removed. Each cone has radius 2 and height 2. Find the volume of the comparison solid.

ANSWER _____

- 4** Solid A and Solid B are both 9 units tall. Their cross sections have equal area at every matching height. If Solid A has volume 72 cubic units, find the volume of Solid B.

ANSWER _____

- 5** A sphere is compared to a cylinder of radius 7 and height 14 with two cones removed. Each cone has radius 7 and height 7. Find the sphere's volume.

ANSWER _____

APPLY IT Show your work

- 1** An esports tournament orders a solid spherical trophy with radius 9 cm. Find the amount of material in the trophy, in cubic centimeters.

WORK SPACE

ANSWER _____

- 2** A robotics club prints a solid spherical sensor cover with radius 5 cm. Use the cylinder-minus-two-cones comparison to find its volume.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Let a cylinder have radius r and height $2r$. Remove two cones, each with radius r and height r , with their tips meeting at the cylinder's center. In 3 sentences, use cross sections and Cavalieri's principle to justify the sphere volume formula.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Find the volume of a sphere with radius 3 units.	36π cubic units
2	Find the volume of a sphere with diameter 8 units.	$256/3 \pi$ cubic units
3	A comparison solid is a cylinder of radius 2 and height 4 with two cones removed. Each cone has radius 2 and height 2. Find the volume of the comparison solid.	$32/3 \pi$ cubic units
4	Solid A and Solid B are both 9 units tall. Their cross sections have equal area at every matching height. If Solid A has volume 72 cubic units, find the volume of Solid B.	72 cubic units
5	A sphere is compared to a cylinder of radius 7 and height 14 with two cones removed. Each cone has radius 7 and height 7. Find the sphere's volume.	$1372/3 \pi$ cubic units

PART 2 • APPLY IT

- 972π cubic centimeters** Use the sphere formula, $V = \frac{4}{3} \pi r^3$. Substituting $r = 9$ gives 972π .
- $500/3 \pi$ cubic centimeters** The comparison cylinder has volume 250π , and the two cones have total volume $250/3 \pi$. Subtract to get $500/3 \pi$.

CHALLENGE

At height x from the center, the sphere's slice has area $\pi(r^2 - x^2)$, and the cylinder with the appropriate cone removed has the same area. Because the matching slices have equal area for every height, Cavalieri's principle gives the solids equal volume. The cylinder volume is $2\pi r^3$ and the two cone volumes total $2/3 \pi r^3$, so the sphere volume is $4/3 \pi r^3$.

Accept equivalent exact forms, such as $32\pi/3$ for $32/3 \pi$. Explanations should mention equal matching cross-sectional areas and subtracting two cone volumes from the cylinder volume.