

Circle Equation Lab

Build and decode circle equations on the coordinate plane.

HIGH SCHOOL • HSG-GPE.A.1

CCSS.Math.Content.HSG-GPE.A.1 "Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation."

NAME _____

DATE _____

THE BIG IDEA

A circle with center (h, k) and radius r has equation $(x - h)^2 + (y - k)^2 = r^2$. To find a center and radius from an equation, group the x terms and y terms, complete each square, and rewrite the equation in this form.

WORKED EXAMPLE

Write the equation of the circle with center $(3, -4)$ and radius 6.

1. From the center, the horizontal and vertical distances are $x - 3$ and $y + 4$.
2. Use the Pythagorean Theorem: $(x - 3)^2 + (y + 4)^2 = 6^2$.
3. Square the radius: $(x - 3)^2 + (y + 4)^2 = 36$.

Answer: $(x - 3)^2 + (y + 4)^2 = 36$

PRACTICE 6 problems

- 1** Write the equation of the circle with center $(-1, 2)$ and radius 5.

ANSWER _____

- 2** Write the equation of the circle with center $(0, -7)$ that passes through $(8, -7)$.

ANSWER _____

- 3** Rewrite $x^2 + y^2 + 10x - 14y + 25 = 0$ in circle form. Then give the center and radius.

ANSWER _____

- 4** Rewrite $x^2 + y^2 - 2x + 16y + 1 = 0$ in circle form. Then give the center and radius.

ANSWER _____

- 5** Rewrite $x^2 + y^2 - 24x - 10y = 0$ in circle form. Then give the center and radius.

ANSWER _____

- 6** Rewrite $x^2 + y^2 - 16x + 30y = 0$ in circle form. Then give the center and radius.

ANSWER _____

APPLY IT Show your work

- 1** An esports arena map uses coordinate units. A circular signal range is centered at $(-10, 8)$ and reaches 15 units in every direction. Write an equation for the boundary of the signal range.

WORK SPACE

ANSWER _____

- 2** On a concert stage map, the boundary of a circular spotlight is modeled by $x^2 + y^2 + 18x - 20y + 117 = 0$. Find the spotlight's center and radius.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Without graphing, determine whether $x^2 + y^2 + 20x - 14y + 150 = 0$ represents a real circle. Complete the square and explain your reasoning.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Write the equation of the circle with center $(-1, 2)$ and radius 5.	$(x + 1)^2 + (y - 2)^2 = 25$
2	Write the equation of the circle with center $(0, -7)$ that passes through $(8, -7)$.	$x^2 + (y + 7)^2 = 64$
3	Rewrite $x^2 + y^2 + 10x - 14y + 25 = 0$ in circle form. Then give the center and radius.	$(x + 5)^2 + (y - 7)^2 = 49$; center $(-5, 7)$; radius 7
4	Rewrite $x^2 + y^2 - 2x + 16y + 1 = 0$ in circle form. Then give the center and radius.	$(x - 1)^2 + (y + 8)^2 = 64$; center $(1, -8)$; radius 8
5	Rewrite $x^2 + y^2 - 24x - 10y = 0$ in circle form. Then give the center and radius.	$(x - 12)^2 + (y - 5)^2 = 169$; center $(12, 5)$; radius 13
6	Rewrite $x^2 + y^2 - 16x + 30y = 0$ in circle form. Then give the center and radius.	$(x - 8)^2 + (y + 15)^2 = 289$; center $(8, -15)$; radius 17

PART 2 • APPLY IT

1. $(x + 10)^2 + (y - 8)^2 = 225$ Use the center as $(-10, 8)$, so the coordinate factors are $x + 10$ and $y - 8$. Square the radius, $15^2 = 225$.
2. **Center $(-9, 10)$; radius 8** Complete the squares to rewrite the equation as $(x + 9)^2 + (y - 10)^2 = 64$. The center is $(-9, 10)$, and the radius is $\sqrt{64} = 8$.

CHALLENGE

No real circle. Completing the square gives $(x + 10)^2 + (y - 7)^2 = -1$. A sum of two squares cannot equal -1 for real coordinates.

Accept equivalent circle forms with terms in a different order. For completed-square problems, require both the rewritten equation and the correct center and positive radius.