

Focus to Formula

Write parabola equations from a focus and directrix.

HIGH SCHOOL • HSG-GPE.A.2

CCSS.Math.Content.HSG-GPE.A.2 "Derive the equation of a parabola given a focus and directrix."

NAME _____

DATE _____

THE BIG IDEA

A parabola is the set of points that are the same distance from a focus and a directrix. Find the vertex halfway between the focus and directrix, then let p be the signed distance from the vertex to the focus. Use $(x-h)^2 = 4p(y-k)$ for a vertical parabola and $(y-k)^2 = 4p(x-h)$ for a horizontal parabola.

WORKED EXAMPLE

Write the equation of the parabola with focus $(4, -5)$ and directrix $y = 1$.

1. The vertex is halfway between $y = -5$ and $y = 1$, so the vertex is $(4, -2)$.
2. The signed distance from the vertex to the focus is $p = -3$.
3. Use $(x-h)^2 = 4p(y-k)$: $(x-4)^2 = 4(-3)(y+2)$.

Answer: $(x-4)^2 = -12(y+2)$

PRACTICE 6 problems

- 1** Write the equation of the parabola with focus $(0, 7)$ and directrix $y = -7$.

ANSWER _____

- 2** Write the equation of the parabola with focus $(-8, 2)$ and directrix $y = 10$.

ANSWER _____

- 3** Write the equation of the parabola with focus $(20, -9)$ and directrix $x = 34$.

ANSWER _____

- 4** Write the equation of the parabola with focus $(-17, 15)$ and directrix $x = -29$.

ANSWER _____

- 5** Write the equation of the parabola with focus $(35, 24)$ and directrix $y = 42$.

ANSWER _____

- 6** Write the equation of the parabola with focus $(45, -14)$ and directrix $y = -30$.

ANSWER _____

APPLY IT Show your work

- 1** A streamer is designing a game overlay. The edge of a spotlight is modeled by a parabola with focus $(7, 18)$ and directrix $y = 8$. Write the equation of the spotlight edge.

WORK SPACE

ANSWER _____

- 2** On a robotics team's test grid, a sensor's detection edge is modeled by a parabola with focus $(-21, 26)$ and directrix $x = 9$. Write the equation of the detection edge.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A parabola has focus $(30, -17)$ and directrix $x = -14$. Use squared distances from an arbitrary point (x, y) to derive its equation. Then state whether it opens left or right.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Write the equation of the parabola with focus (0, 7) and directrix $y = -7$.	$x^2 = 28y$
2	Write the equation of the parabola with focus (-8, 2) and directrix $y = 10$.	$(x+8)^2 = -16(y-6)$
3	Write the equation of the parabola with focus (20, -9) and directrix $x = 34$.	$(y+9)^2 = -28(x-27)$
4	Write the equation of the parabola with focus (-17, 15) and directrix $x = -29$.	$(y-15)^2 = 24(x+23)$
5	Write the equation of the parabola with focus (35, 24) and directrix $y = 42$.	$(x-35)^2 = -36(y-33)$
6	Write the equation of the parabola with focus (45, -14) and directrix $y = -30$.	$(x-45)^2 = 32(y+22)$

PART 2 • APPLY IT

1. $(x-7)^2 = 20(y-13)$ The vertex is halfway between the focus and directrix at (7, 13), and $p = 5$. Substitute into the vertical-parabola form.
2. $(y-26)^2 = -60(x+6)$ The vertex is (-6, 26), halfway between $x = -21$ and $x = 9$, and $p = -15$. Substitute into the horizontal-parabola form.

CHALLENGE

$(x-30)^2 + (y+17)^2 = (x+14)^2$, so $(y+17)^2 = 88(x-8)$. The parabola opens right.

Accept equivalent equations, including expanded forms that simplify to the same equation. For the challenge, accept a correct squared-distance setup and the statement that the parabola opens right.