LESSON MAKER PRO

Focus to Formula

Write parabola equations from a focus and directrix.

HIGH SCHOOL · HSG-GPE.A.2

CCSS.Math.Content.HSG-GPE.A.2 "Derive the equation of a parabola given a focus and directrix."

NAME _____ DATE _____

THE BIG IDEA

A parabola is the set of points that are the same distance from a focus and a directrix. Find the vertex halfway between the focus and directrix, then let p be the signed distance from the vertex to the focus. Use $(x-h)^2 = 4p(y-k)$ for a vertical parabola and $(y-k)^2 = 4p(x-h)$ for a horizontal parabola.

WORKED EXAMPLE

Write the equation of the parabola with focus (4, -5) and directrix y = 1.

1. The vertex is halfway between y = -5 and y = 1, so the vertex is (4, -2).
2. The signed distance from the vertex to the focus is p = -3.
3. Use $(x-h)^2 = 4p(y-k)$: $(x-4)^2 = 4(-3)(y+2)$.

Answer: $(x-4)^2 = -12(y+2)$

PRACTICE 6 problems

1 Write the equation of the parabola with focus (0, 7) and directrix y = -7.

ANSWER _____

2 Write the equation of the parabola with focus (-8, 2) and directrix y = 10.

ANSWER _____

3 Write the equation of the parabola with focus (20, -9) and directrix x = 34.

ANSWER _____

4 Write the equation of the parabola with focus (-17, 15) and directrix x = -29.

ANSWER _____

5 Write the equation of the parabola with focus (35, 24) and directrix y = 42.

ANSWER _____

6 Write the equation of the parabola with focus (45, -14) and directrix y = -30.

ANSWER _____

APPLY IT Show your work

1 A streamer is designing a game overlay. The edge of a spotlight is modeled by a parabola with focus (7, 18) and directrix y = 8. Write the equation of the spotlight edge.

WORK SPACE

ANSWER _____

2 On a robotics team's test grid, a sensor's detection edge is modeled by a parabola with focus (-21, 26) and directrix x = 9. Write the equation of the detection edge.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A parabola has focus (30, -17) and directrix x = -14. Use squared distances from an arbitrary point (x, y) to derive its equation. Then state whether it opens left or right.

WORK SPACE AND ANSWER

Focus to Formula

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Write the equation of the parabola with focus (0, 7) and directrix y = -7.	**$x^2 = 28y$**
2	Write the equation of the parabola with focus (-8, 2) and directrix y = 10.	**$(x+8)^2 = -16(y-6)$**
3	Write the equation of the parabola with focus (20, -9) and directrix x = 34.	**$(y+9)^2 = -28(x-27)$**
4	Write the equation of the parabola with focus (-17, 15) and directrix x = -29.	**$(y-15)^2 = 24(x+23)$**
5	Write the equation of the parabola with focus (35, 24) and directrix y = 42.	**$(x-35)^2 = -36(y-33)$**
6	Write the equation of the parabola with focus (45, -14) and directrix y = -30.	**$(x-45)^2 = 32(y+22)$**

PART 2 · APPLY IT

1. **$(x-7)^2 = 20(y-13)$** The vertex is halfway between the focus and directrix at (7, 13), and p = 5. Substitute into the vertical-parabola form.
2. **$(y-26)^2 = -60(x+6)$** The vertex is (-6, 26), halfway between x = -21 and x = 9, and p = -15. Substitute into the horizontal-parabola form.

CHALLENGE

$(x-30)^2 + (y+17)^2 = (x+14)^2$, so $(y+17)^2 = 88(x-8)$. The parabola opens right.

Accept equivalent equations, including expanded forms that simplify to the same equation. For the challenge, accept a correct squared-distance setup and the statement that the parabola opens right.