

Focus to Formula

Build conic equations from distance rules.

HIGH SCHOOL • HSG-GPE.A.3

CCSS.Math.Content.HSG-GPE.A.3 "(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant."

NAME _____

DATE _____

DISTANCE CREATES THE SHAPE

Find the center, then let c be the distance from the center to either focus. For an ellipse, the constant sum is $2a$ and $b^2 = a^2 - c^2$. For a hyperbola, the constant positive difference is $2a$ and $b^2 = c^2 - a^2$; put a^2 under the coordinate along the foci and use a minus sign between the fractions.

WORKED EXAMPLE

Find the equation of the ellipse with foci $(11, 4)$ and $(11, 18)$ if the sum of the distances to the foci is 30.

1. The center is $(11, 11)$, $c = 7$, and the major axis is vertical.
2. $2a = 30$, so $a = 15$.
3. $b^2 = a^2 - c^2 = 225 - 49 = 176$.
4. Use y for the vertical major-axis term: $(x - 11)^2/176 + (y - 11)^2/225 = 1$.

Answer: $(x - 11)^2/176 + (y - 11)^2/225 = 1$

PRACTICE 6 problems

- 1** Find the equation of the ellipse with foci $(-5, 0)$ and $(5, 0)$ if the sum of the distances to the foci is 26.

ANSWER _____

- 2** Find the equation of the ellipse with foci $(2, -6)$ and $(2, 6)$ if the sum of the distances to the foci is 20.

ANSWER _____

- 3** Find the equation of the hyperbola with foci $(-10, 1)$ and $(10, 1)$ if the positive difference of the distances to the foci is 12.

ANSWER _____

- 4** Find the equation of the hyperbola with foci $(-3, -9)$ and $(-3, 7)$ if the positive difference of the distances to the foci is 10.

ANSWER _____

- 5** Find the equation of the ellipse with foci $(14, -6)$ and $(14, 6)$ if the sum of the distances to the foci is 34.

ANSWER _____

- 6** Find the equation of the hyperbola with foci $(6, -2)$ and $(6, 12)$ if the positive difference of the distances to the foci is 10.

ANSWER _____

APPLY IT Show your work

- 1** A track team designs a training route on a coordinate grid. The route is modeled by all points whose distances from course markers at $(0, 0)$ and $(10, 0)$ add to 26 meters. Write the equation of the route.

WORK SPACE

ANSWER _____

- 2** A robotics team maps locations where the absolute difference between distances to two signal receivers at $(-13, 2)$ and $(13, 2)$ is 10 meters. Write the equation of the path.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A video-game designer models an oval arena boundary as an ellipse with foci $(0, -12)$ and $(0, 12)$. The boundary passes through $(5, 0)$. Find the constant sum of distances and the equation of the ellipse.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Find the equation of the ellipse with foci $(-5, 0)$ and $(5, 0)$ if the sum of the distances to the foci is 26.	$x^2/169 + y^2/144 = 1$
2	Find the equation of the ellipse with foci $(2, -6)$ and $(2, 6)$ if the sum of the distances to the foci is 20.	$(x - 2)^2/64 + y^2/100 = 1$
3	Find the equation of the hyperbola with foci $(-10, 1)$ and $(10, 1)$ if the positive difference of the distances to the foci is 12.	$x^2/36 - (y - 1)^2/64 = 1$
4	Find the equation of the hyperbola with foci $(-3, -9)$ and $(-3, 7)$ if the positive difference of the distances to the foci is 10.	$(y + 1)^2/25 - (x + 3)^2/39 = 1$
5	Find the equation of the ellipse with foci $(14, -6)$ and $(14, 6)$ if the sum of the distances to the foci is 34.	$(x - 14)^2/253 + y^2/289 = 1$
6	Find the equation of the hyperbola with foci $(6, -2)$ and $(6, 12)$ if the positive difference of the distances to the foci is 10.	$(y - 5)^2/25 - (x - 6)^2/24 = 1$

PART 2 • APPLY IT

1. $(x - 5)^2/169 + y^2/144 = 1$ The center is $(5, 0)$, $c = 5$, and $2a = 26$, so $a = 13$. Then $b^2 = 13^2 - 5^2 = 144$.
2. $x^2/25 - (y - 2)^2/144 = 1$ The center is $(0, 2)$, $c = 13$, and $2a = 10$, so $a = 5$. Then $b^2 = 13^2 - 5^2 = 144$.

CHALLENGE

The constant sum is 26, and the equation is $x^2/25 + y^2/169 = 1$.

Accept equivalent equations with terms written in either order. For hyperbolas, accept an equivalent form only if the positive term is along the axis containing the foci.