

Coordinate Case Files

Use slopes and distances to test geometric claims.

HIGH SCHOOL • HSG-GPE.B.4

CCSS.Math.Content.HSG-GPE.B.4 "Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1..."

NAME _____

DATE _____

COORDINATE PROOF TOOLS

Use slope to test parallel or perpendicular sides. Equal slopes show parallel lines, and a horizontal line and a vertical line are perpendicular. Use squared distance, $(x_2 - x_1)^2 + (y_2 - y_1)^2$, to compare side lengths or radii without finding square roots.

WORKED EXAMPLE

Prove or disprove that P(11,17), Q(23,17), R(23,29), and S(11,29), in order, form a square.

1. PQ is horizontal because both y-coordinates are 17. QR is vertical because both x-coordinates are 23.
2. RS is horizontal and SP is vertical, so opposite sides are parallel and adjacent sides are perpendicular.
3. $PQ = 12$ and $QR = 12$, so adjacent sides are equal.

Answer: PQRS is a square, so it is also a rectangle.

PRACTICE 6 problems

- 1** Do A(-2,1), B(2,1), and C(2,4) form a right triangle? Use slopes or horizontal and vertical lines to justify your answer.

ANSWER _____

- 2** Prove or disprove that triangle J(-4,-1), K(0,3), and L(4,-1) is isosceles.

ANSWER _____

- 3** Prove or disprove that A(-3,0), B(1,0), C(1,2), and D(-3,2), in order, form a rectangle.

ANSWER _____

- 4** A circle is centered at the origin and contains (0,5). Does the point (3,4) lie on the circle? Show a coordinate calculation.

ANSWER _____

- 5** A circle has center (2,-1) and contains the point (5,3). Does the point (6,2) lie on the same circle?

ANSWER _____

- 6** Prove or disprove that W(-2,-3), X(3,-3), Y(5,1), and Z(0,1), in order, form a parallelogram.

ANSWER _____

APPLY IT Show your work

- 1** To plan a school esports event, a student team marks a viewing zone on a coordinate grid. Its corners, in order, are $A(-5,1)$, $B(3,1)$, $C(3,6)$, and $D(-5,6)$. Use coordinates to prove or disprove that the viewing zone is a rectangle.

WORK SPACE

ANSWER _____

- 2** A drone club maps a circular safety boundary with center $(-2,3)$. A point on the boundary is $(1,7)$. The club wants to know whether waypoint $(2,6)$ is also on the boundary. Use coordinates to decide.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Explain why $A(-1,1)$, $B(2,3)$, $C(4,0)$, and $D(1,-2)$, in order, form a square, not only a rectangle.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Do A(-2,1), B(2,1), and C(2,4) form a right triangle? Use slopes or horizontal and vertical lines to justify your answer.	Yes. AB is horizontal and BC is vertical, so angle ABC is a right angle.
2	Prove or disprove that triangle J(-4,-1), K(0,3), and L(4,-1) is isosceles.	Yes. $JK^2 = 32$ and $KL^2 = 32$, so $JK = KL$. Triangle JKL is isosceles.
3	Prove or disprove that A(-3,0), B(1,0), C(1,2), and D(-3,2), in order, form a rectangle.	Yes. AB and CD have slope 0, while BC and DA are vertical. Adjacent sides are perpendicular, so ABCD is a rectangle.
4	A circle is centered at the origin and contains (0,5). Does the point (3,4) lie on the circle? Show a coordinate calculation.	Yes. $3^2 + 4^2 = 25$, and $5^2 = 25$, so (3,4) lies on the circle.
5	A circle has center (2,-1) and contains the point (5,3). Does the point (6,2) lie on the same circle?	Yes. The squared distance from (2,-1) to (5,3) is $3^2 + 4^2 = 25$. The squared distance from (2,-1) to (6,2) is $4^2 + 3^2 = 25$.
6	Prove or disprove that W(-2,-3), X(3,-3), Y(5,1), and Z(0,1), in order, form a parallelogram.	Yes. Slope WX = slope YZ = 0, and slope XY = slope ZW = 2. Both pairs of opposite sides are parallel, so WXYZ is a parallelogram.

PART 2 • APPLY IT

- The viewing zone is a rectangle.** AB and CD are horizontal, while BC and DA are vertical. Therefore, opposite sides are parallel and adjacent sides are perpendicular.
- Yes. Waypoint (2,6) lies on the safety boundary.** The squared distance from (-2,3) to (1,7) is $3^2 + 4^2 = 25$. The squared distance from (-2,3) to (2,6) is $4^2 + 3^2 = 25$, so both points are the same distance from the center.

CHALLENGE

AB and CD have slope $\frac{2}{3}$, while BC and DA have slope $-\frac{3}{2}$. The adjacent slopes multiply to -1, so the angles are right angles. Also, $AB^2 = BC^2 = CD^2 = DA^2 = 13$. The figure has four right angles and four equal sides, so ABCD is a square.

Accept slope comparisons, squared-distance calculations, or clear horizontal and vertical line reasoning. For circle questions, accept any correct comparison showing that the point and the given boundary point are the same distance from the center.