

Triangle Side Quest

Use exact values with the Laws of Sines and Cosines.

HIGH SCHOOL • HSG-SRT.D.10

CCSS.Math.Content.HSG-SRT.D.10 "(+) Prove the Laws of Sines and Cosines and use them to solve problems."

NAME _____

DATE _____

CHOOSE THE RIGHT LAW

In triangle ABC, lowercase side a is opposite uppercase angle A , and similarly for b and c . The Law of Sines is $a/\sin A = b/\sin B = c/\sin C$, so use it when an opposite side and angle are known. The Law of Cosines is $a^2 = b^2 + c^2 - 2bc \cos A$, so use a matching version when you know two sides and their included angle, or all three sides.

WORKED EXAMPLE

In triangle ABC, $b = 5$, $c = 7$, and $A = 120$ degrees. Find a .

1. $a^2 = b^2 + c^2 - 2bc \cos A$
2. $a^2 = 5^2 + 7^2 - 2(5)(7)\cos(120 \text{ degrees})$
3. $a^2 = 25 + 49 + 35$
4. $a^2 = 109$

Answer: $a = \sqrt{109}$

PRACTICE 6 problems

- 1** In triangle ABC, $b = 9$, $c = 11$, and $A = 60$ degrees. Find a .

ANSWER _____

- 2** In triangle ABC, $a = 12$, $b = 16$, and $C = 90$ degrees. Find c .

ANSWER _____

- 3** In triangle ABC, $A = 30$ degrees, $a = 8$, and $B = 90$ degrees. Find b .

ANSWER _____

- 4** In triangle ABC, $A = 45$ degrees, $B = 45$ degrees, and $a = 9$. Find b .

ANSWER _____

- 5** In triangle ABC, $a = 8$, $b = 8$, and $C = 60$ degrees. Find c .

ANSWER _____

- 6** In triangle ABC, $a = 12$, $b = 13$, and $c = 17$. Find the exact value of $\cos C$.

ANSWER _____

APPLY IT Show your work

- 1** A student in the school film club marks two camera positions 8 meters and 10 meters from the same stage entrance. The angle between the paths from the entrance to the two positions is 60 degrees. How far apart are the camera positions? Give an exact answer.

WORK SPACE

ANSWER _____

- 2** A student maps a triangular cross-country practice course. At checkpoint A, the angle is 30 degrees, and at checkpoint B, the angle is 45 degrees. The side opposite angle A is 6 kilometers long. What is the exact length of the side opposite angle B?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

In triangle ABC, $b = 10$, $c = 14$, and $A = 60$ degrees. Find the exact value of $\cos B$. First find side a , then use the Law of Cosines again.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	In triangle ABC, $b = 9$, $c = 11$, and $A = 60$ degrees. Find a .	$a = \sqrt{103}$
2	In triangle ABC, $a = 12$, $b = 16$, and $C = 90$ degrees. Find c .	$c = 20$
3	In triangle ABC, $A = 30$ degrees, $a = 8$, and $B = 90$ degrees. Find b .	$b = 16$
4	In triangle ABC, $A = 45$ degrees, $B = 45$ degrees, and $a = 9$. Find b .	$b = 9$
5	In triangle ABC, $a = 8$, $b = 8$, and $C = 60$ degrees. Find c .	$c = 8$
6	In triangle ABC, $a = 12$, $b = 13$, and $c = 17$. Find the exact value of $\cos C$.	$\cos C = 1/13$

PART 2 • APPLY IT

- $2\sqrt{21}$ meters** Let d be the distance between the positions. $d^2 = 8^2 + 10^2 - 2(8)(10)\cos(60 \text{ degrees}) = 84$, so $d = 2\sqrt{21}$.
- $6\sqrt{2}$ kilometers** Use $b/\sin(45 \text{ degrees}) = 6/\sin(30 \text{ degrees})$. This gives $b = 6\sqrt{2}$.

CHALLENGE

$a = 2\sqrt{39}$, and $\cos B = 3\sqrt{39}/26$

Accept equivalent exact radical forms, including $9/(2\sqrt{39})$ for the challenge value of $\cos B$. Require positive side lengths and do not accept decimal approximations when an exact answer is requested.