

Triangle Area Shortcut

Use an altitude and sine to find exact triangle areas.

HIGH SCHOOL • HSG-SRT.D.9

CCSS.Math.Content.HSG-SRT.D.9 "(+) Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side."

NAME _____

DATE _____

BUILD THE FORMULA

When sides a and b meet at angle C , use side a as the base. Draw an altitude from the endpoint of b to the line containing a . The altitude is $h = b \sin(C)$, so $A = \frac{1}{2} ah = \frac{1}{2} ab \sin(C)$.

WORKED EXAMPLE

A triangle has $a = 13$, $b = 8$, and $C = 90$ degrees. Find its area.

1. Draw the altitude h from the endpoint of b to side a .
2. $h = b \sin(C) = 8 \sin(90 \text{ degrees}) = 8$.
3. $A = \frac{1}{2} ah = \frac{1}{2}(13)(8)$.
4. $A = 52$.

Answer: 52 square units

PRACTICE 6 problems

- 1** A triangle has $a = 9$, $b = 12$, and $C = 45$ degrees. Find its area in exact form.

ANSWER _____

- 2** A triangle has $a = 7$, $b = 6$, and $C = 30$ degrees. Find its area in exact form.

ANSWER _____

- 3** A triangle has $a = 14$, $b = 5$, and $C = 60$ degrees. Find its area in exact form.

ANSWER _____

- 4** A triangle has $a = 11$, $b = 4$, and $C = 15$ degrees. Find its area in exact form.

ANSWER _____

- 5** A triangle has $a = 2$, $b = 15$, and $C = 60$ degrees. First find $h = b \sin(C)$, then find the area.

ANSWER _____

- 6** A triangle has $a = 16$, $b = 3$, and $C = 45$ degrees. Find its area in exact form.

ANSWER _____

APPLY IT Show your work

- 1** A school club is cutting a triangular fabric panel for a concert backdrop. Two sides that meet are 15 cm and 6 cm, and their included angle is 60 degrees. What is the panel's area in exact form?

WORK SPACE

ANSWER _____

- 2** A student designs a triangular graphic for a gaming event poster. Two sides of the graphic are 18 inches and 5 inches, with an included angle of 45 degrees. Find the exact area.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A triangle has sides a and b enclosing angle C . Sketch an altitude h from the endpoint of b to the line containing side a . Explain why $A = \frac{1}{2} ab \sin(C)$. Include an equation for h and an equation for area.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	A triangle has $a = 9$, $b = 12$, and $C = 45$ degrees. Find its area in exact form.	$27\sqrt{2}$ square units
2	A triangle has $a = 7$, $b = 6$, and $C = 30$ degrees. Find its area in exact form.	$21/2$ square units
3	A triangle has $a = 14$, $b = 5$, and $C = 60$ degrees. Find its area in exact form.	$35\sqrt{3}/2$ square units
4	A triangle has $a = 11$, $b = 4$, and $C = 15$ degrees. Find its area in exact form.	$11(\sqrt{6} - \sqrt{2})/2$ square units
5	A triangle has $a = 2$, $b = 15$, and $C = 60$ degrees. First find $h = b \sin(C)$, then find the area.	$h = 15\sqrt{3}/2$ units; area = $15\sqrt{3}/2$ square units
6	A triangle has $a = 16$, $b = 3$, and $C = 45$ degrees. Find its area in exact form.	$12\sqrt{2}$ square units

PART 2 • APPLY IT

- $45\sqrt{3}/2$ square centimeters** Use $A = 1/2 ab \sin(C)$. $A = 1/2(15)(6)\sin(60 \text{ degrees}) = 45\sqrt{3}/2$ square centimeters.
- $45\sqrt{2}/2$ square inches** Use $A = 1/2 ab \sin(C)$. $A = 1/2(18)(5)\sin(45 \text{ degrees}) = 45\sqrt{2}/2$ square inches.

CHALLENGE

The altitude is $h = b \sin(C)$ because $\sin(C) = h/b$. Then $A = 1/2 ah = 1/2 a(b \sin(C)) = 1/2 ab \sin(C)$.

Accept equivalent exact radical forms, such as $27\sqrt{2}$ or $27\sqrt{2}/1$. For the challenge, accept a correct derivation that identifies $h = b \sin(C)$ and substitutes it into $A = 1/2 ah$.