

Complex Number Moves

Conjugates, moduli, and quotients in $a + bi$ form.

HIGH SCHOOL • HSN-CN.A.3

CCSS.Math.Content.HSN-CN.A.3 "(+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers."

NAME _____

DATE _____

THE BIG IDEA

For $z = a + bi$, the conjugate is $a - bi$. The modulus of $a + bi$ is $\sqrt{a^2 + b^2}$. To divide complex numbers, multiply the numerator and denominator by the conjugate of the denominator so the denominator becomes real.

WORKED EXAMPLE

Simplify $(11 + 13i)/(5 - 4i)$.

1. The conjugate of $5 - 4i$ is $5 + 4i$.
2. Multiply the numerator and denominator by $5 + 4i$.
3. $[(11 + 13i)(5 + 4i)] / [(5 - 4i)(5 + 4i)] = (3 + 109i)/41$.

Answer: $3/41 + 109/41 i$

PRACTICE 6 problems

1 Find the conjugate of $-6 + 8i$.

ANSWER _____

2 Find the conjugate of $7 - 2i$.

ANSWER _____

3 Find the modulus of $9 - i$.

ANSWER _____

4 Find the modulus of $-8 + 15i$.

ANSWER _____

5 Simplify $(17 + 6i)/(2 + i)$. Write your answer in $a + bi$ form.

ANSWER _____

6 Simplify $(21 + 12i)/(1 + 2i)$. Write your answer in $a + bi$ form.

ANSWER _____

APPLY IT Show your work

- 1** A student uses a music-mixing app that models a track's signal as $1 + 8i$. A calibration filter is $7 - 2i$, and the normalized signal is $\text{signal}/\text{filter}$. Find the normalized signal in $a + bi$ form.

WORK SPACE

ANSWER _____

- 2** In a robotics competition, a robot's position correction is modeled by $-8 + 15i$ centimeters, where the real and imaginary parts give perpendicular directions. The size of the correction is its modulus. Find the size of the correction.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Without using decimals, simplify $(48 - 29i)/(6 + i)$. Then find the modulus of your quotient.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Find the conjugate of $-6 + 8i$.	$-6 - 8i$
2	Find the conjugate of $7 - 2i$.	$7 + 2i$
3	Find the modulus of $9 - i$.	$\sqrt{82}$
4	Find the modulus of $-8 + 15i$.	17
5	Simplify $(17 + 6i)/(2 + i)$. Write your answer in $a + bi$ form.	$8 - i$
6	Simplify $(21 + 12i)/(1 + 2i)$. Write your answer in $a + bi$ form.	$9 - 6i$

PART 2 • APPLY IT

1. **$-9/53 + 58/53 i$** Multiply the numerator and denominator by $7 + 2i$, the conjugate of the denominator. Simplify the resulting real denominator of 53.
2. **17 centimeters** Find $\sqrt{(-8)^2 + 15^2}$. This is $\sqrt{289}$, which equals 17.

CHALLENGE

Quotient: $7 - 6i$; modulus: $\sqrt{85}$

Accept equivalent quotient forms, including answers with separate fractional real and imaginary parts. For moduli, accept an unsimplified radical only when it is mathematically equivalent to the listed answer.