LESSON MAKER PRO

Complex Plane Switch

Move between rectangular and polar forms.

HIGH SCHOOL · HSN-CN.B.4

CCSS.Math.Content.HSN-CN.B.4 "(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number."

NAME _____ DATE _____

TWO FORMS, ONE POINT

A complex number a + bi has rectangular coordinates (a, b) on the complex plane. In polar form, z = r(cos theta + i sin theta), where r is the distance from the origin and theta is the angle from the positive real axis. The forms name the same point because a = r cos theta and b = r sin theta.

WORKED EXAMPLE

Write z = 7 + 7i in polar form with r > 0 and 0° <= theta < 360°.

1. a = 7 and b = 7.
2. r = sqrt(7^2 + 7^2) = 7sqrt2.
3. tan theta = 7/7 = 1, so theta = 45° in Quadrant I.
4. z = 7sqrt2(cos 45° + i sin 45°).

Answer: 7sqrt2(cos 45° + i sin 45°)

PRACTICE 6 problems

1 Write z = -sqrt3 + i in polar form with r > 0 and 0° <= theta < 360°.

ANSWER _____

2 Write 4(cos 60° + i sin 60°) in rectangular form.

ANSWER _____

3 Give the rectangular coordinates and rectangular form of 3(cos 180° + i sin 180°).

ANSWER _____

4 Write z = -2 - 2i in polar form with r > 0 and 0° <= theta < 360°.

ANSWER _____

5 Write 5(cos 270° + i sin 270°) in rectangular form.

ANSWER _____

6 The polar coordinates of a complex number are (2, 300°). Write the number in rectangular form and give its rectangular coordinates.

ANSWER _____

APPLY IT Show your work

1 A video game uses the complex number 4(cos 150° + i sin 150°) to represent a character's movement from the origin. The real part is the horizontal change, and the imaginary part is the vertical change. Write the movement in rectangular form.

WORK SPACE

ANSWER _____

2 The audio club models a sound setting with the complex number -1 - sqrt3i. Write this setting in polar form with r > 0 and 0° <= theta < 360°.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A student says that -2 + 2sqrt3i and 4(cos 120° + i sin 120°) are different because one is rectangular and one is polar. Show that the two forms are equal. Then give one other polar form for the same number that has a positive radius.

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ANSWER KEY

Complex Plane Switch

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Write z = -sqrt3 + i in polar form with r > 0 and 0° <= theta < 360°.	**2(cos 150° + i sin 150°)**
2	Write 4(cos 60° + i sin 60°) in rectangular form.	**2 + 2sqrt3i**
3	Give the rectangular coordinates and rectangular form of 3(cos 180° + i sin 180°).	**(-3, 0); -3 + 0i**
4	Write z = -2 - 2i in polar form with r > 0 and 0° <= theta < 360°.	**2sqrt2(cos 225° + i sin 225°)**
5	Write 5(cos 270° + i sin 270°) in rectangular form.	**-5i**
6	The polar coordinates of a complex number are (2, 300°). Write the number in rectangular form and give its rectangular coordinates.	**1 - sqrt3i; (1, -sqrt3)**

PART 2 · APPLY IT

1. **-2sqrt3 + 2i** Use r cos theta for the real part and r sin theta for the imaginary part. Since cos 150° = -sqrt3/2 and sin 150° = 1/2, the components are -2sqrt3 and 2.
2. **2(cos 240° + i sin 240°)** The magnitude is sqrt((-1)^2 + (-sqrt3)^2) = 2. Both components are negative, so the point is in Quadrant III, at 240°.

CHALLENGE

4(cos 120° + i sin 120°) = 4(-1/2 + i(sqrt3/2)) = -2 + 2sqrt3i, so both forms describe the same complex number. One other polar form is 4(cos 480° + i sin 480°).

Accept polar angles that differ from the listed angle by any multiple of 360° when the radius is positive. Accept rectangular forms with or without an explicit 0i term.