LESSON MAKER PRO

Complex Plane Switch

Move between rectangular and polar forms.

HIGH SCHOOL · HSN-CN.B.4

CCSS.Math.Content.HSN-CN.B.4 "(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number."

NAME _____ DATE _____

TWO FORMS, ONE POINT

A complex number a + bi matches the point (a, b) on the complex plane. In polar form, r(cos θ + i sin θ), r is the distance from the origin and θ is the counterclockwise angle from the positive real axis. Both forms represent the same number because they locate the same point.

WORKED EXAMPLE

Write z = -11 + 11i in polar form.

1. The point is (-11, 11).
2. r = sqrt((-11)^2 + 11^2) = sqrt(242) = 11sqrt(2).
3. The point is in Quadrant II with a reference angle of 45°, so θ = 135°.
4. Write the polar form using r and θ.

Answer: 11sqrt(2)(cos 135° + i sin 135°)

PRACTICE 6 problems

1 Write z = 3 + 3i in polar form.

ANSWER _____

2 Write z = -8i in polar form.

ANSWER _____

3 Write 9(cos 180° + i sin 180°) in rectangular form.

ANSWER _____

4 Write 10(cos 60° + i sin 60°) in rectangular form.

ANSWER _____

5 Write z = -3 - 3sqrt(3)i in polar form.

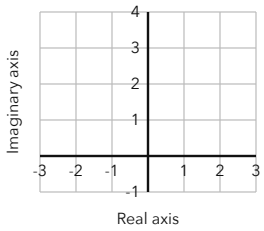
ANSWER _____

6 Show that 2(cos 60° + i sin 60°) and 1 + sqrt(3)i represent the same complex number.

ANSWER _____

APPLY IT Show your work

1 A student coding a game represents a sprite's displacement as z = -2 + 2sqrt(3)i. Plot the displacement on the complex-plane grid. Then write z in polar form with an angle from 0° to 360°.



ANSWER _____

2 An audio club models a filter setting by z = 12(cos 30° + i sin 30°). Write the setting in rectangular form. Then state its real component and imaginary coefficient.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A student claims that -sqrt(3) + i = 2(cos 30° + i sin 30°). Is the claim correct? Explain the error and write the correct polar form with an angle from 0° to 360°.

WORK SPACE AND ANSWER

ANSWER KEY # Complex Plane Switch

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Write z = 3 + 3i in polar form.	**3sqrt(2)(cos 45° + i sin 45°)**
2	Write z = -8i in polar form.	**8(cos 270° + i sin 270°)**
3	Write 9(cos 180° + i sin 180°) in rectangular form.	**-9 + 0i**
4	Write 10(cos 60° + i sin 60°) in rectangular form.	**5 + 5sqrt(3)i**
5	Write z = -3 - 3sqrt(3)i in polar form.	**6(cos 240° + i sin 240°)**
6	Show that 2(cos 60° + i sin 60°) and 1 + sqrt(3)i represent the same complex number.	**2(cos 60° + i sin 60°) = 2(1/2 + i(sqrt(3)/2)) = 1 + sqrt(3)i, so the forms represent the same number.**

PART 2 · APPLY IT

1. **Plot (-2, 2sqrt(3)). Polar form: 4(cos 120° + i sin 120°).** Use the rectangular coordinates (-2, 2sqrt(3)). The distance is sqrt(4 + 12) = 4, and the Quadrant II angle is 120°.
2. **6sqrt(3) + 6i; real component: 6sqrt(3), imaginary coefficient: 6.** Multiply 12 by cos 30° = sqrt(3)/2 and by sin 30° = 1/2. This gives 6sqrt(3) + 6i.

CHALLENGE

No. At 30°, the cosine is positive, so 2(cos 30° + i sin 30°) = sqrt(3) + i. The correct form is 2(cos 150° + i sin 150°), because cos 150° = -sqrt(3)/2 and sin 150° = 1/2.

Accept equivalent polar angles that differ by multiples of 360°. Accept -9 as equivalent to -9 + 0i in rectangular form.