LESSON MAKER PRO

Complex Plane Moves

Use points, vectors, and rotations to compute with complex numbers.

HIGH SCHOOL · HSN-CN.B.5

CCSS.Math.Content.HSN-CN.B.5 "(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example..."

NAME _____ DATE _____

GEOMETRY OF COMPLEX NUMBERS

Treat a + bi as the point (a, b) on the complex plane. Add or subtract by combining horizontal and vertical moves, and reflect across the real axis to find a conjugate. When you multiply, multiply the moduli and add the arguments.

WORKED EXAMPLE

Find $(-2 + 2i)^2$ by using modulus and argument.

1. The modulus of -2 + 2i is $\text{sqrt}((-2)^2 + 2^2) = 2\text{sqrt}2$, and its argument is 135°.
2. Squaring multiplies the modulus by itself: $(2\text{sqrt}2)^2 = 8$.
3. Squaring adds the arguments: 135° + 135° = 270°.
4. A point with modulus 8 and argument 270° is -8i.

Answer: -8i

PRACTICE 6 problems

1 Let z = -4 + 3i and w = 2 - 5i. Find z + w and give its point on the complex plane.

ANSWER _____

2 Find (5 + 2i) - (-1 + 6i).

ANSWER _____

3 Find the conjugate of -3 - 7i. Give the reflected point on the complex plane.

ANSWER _____

4 Find (2 + i)(3 - 2i). Give the product as a complex number and as a point.

ANSWER _____

5 A complex number has modulus 5 and argument 150°. What are the modulus and argument of its conjugate? Use an argument between 0° and 360°.

ANSWER _____

6 Use modulus and argument to find $(-\text{sqrt}3 + i)^3$.

ANSWER _____

APPLY IT Show your work

1 In a video game, a character's first movement is represented by -3 + 4i, and the second movement is represented by 5 + 2i. What complex number represents the total movement, and what is the character's final point if the character started at the origin?

WORK SPACE

ANSWER _____

2 A digital art app represents a point by sqrt3 + i. The app rotates the point 120° counterclockwise by multiplying it by -1/2 + (sqrt3/2)i. Find the new complex number.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Let z = -5 + 12i. Without using FOIL, use modulus and argument to find z times its conjugate. Explain why the result must be a positive real number.

WORK SPACE AND ANSWER

ANSWER KEY

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Let z = -4 + 3i and w = 2 - 5i. Find z + w and give its point on the complex plane.	**z + w = -2 - 2i, point (-2, -2)**
2	Find (5 + 2i) - (-1 + 6i).	**6 - 4i**
3	Find the conjugate of -3 - 7i. Give the reflected point on the complex plane.	**-3 + 7i, point (-3, 7)**
4	Find (2 + i)(3 - 2i). Give the product as a complex number and as a point.	**8 - i, point (8, -1)**
5	A complex number has modulus 5 and argument 150°. What are the modulus and argument of its conjugate? Use an argument between 0° and 360°.	**modulus 5, argument 210°**
6	Use modulus and argument to find $(-\text{sqrt}3 + i)^3$.	**8i**

PART 2 · APPLY IT

1. **2 + 6i, final point (2, 6)** Add the real parts and the imaginary parts: (-3 + 5) + (4 + 2)i = 2 + 6i. The real part is the horizontal coordinate and the imaginary part is the vertical coordinate.
2. **-sqrt3 + i** Multiply the two complex numbers and combine like terms. The result is -sqrt3 + i, which also has the original modulus and an argument increased by 120°.

CHALLENGE

z times conjugate(z) = 169. The modulus of z is 13, so the product has modulus 13 times 13 = 169. The conjugate reflects z across the real axis, so its argument is the opposite of z's argument. The arguments add to 0°, placing the product on the positive real axis.

Accept equivalent arguments that differ by multiples of 360°, such as -150° for 210°. Accept correctly simplified equivalent complex-number forms and coordinate notation with or without parentheses.