

# Complex Plane Paths

*Find distances and midpoints with complex numbers.*

**HIGH SCHOOL · HSN-CN.B.6**

**CCSS.Math.Content.HSN-CN.B.6** "(+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints."

NAME \_\_\_\_\_

DATE \_\_\_\_\_

## DISTANCE AND MIDPOINT RULES

For  $z=a+bi$  and  $w=c+di$ , the distance between them is  $|w-z|=\sqrt{(c-a)^2+(d-b)^2}$ . The midpoint is  $(z+w)/2$ , so average the real parts and average the imaginary parts.

## WORKED EXAMPLE

Find the distance between  $z=7+5i$  and  $w=10+i$ .

1. Subtract:  $w-z=(10+i)-(7+5i)$ .
2. Combine parts:  $w-z=3-4i$ .
3. Find the modulus:  $|3-4i|=\sqrt{3^2+(-4)^2}=\sqrt{25}=5$ .

**Answer: 5**

## PRACTICE 6 problems

- 1** Find the distance between  $z=-2+3i$  and  $w=4+3i$ .

**ANSWER** \_\_\_\_\_

- 2** Find the distance between  $z=-1-2i$  and  $w=2+2i$ .

**ANSWER** \_\_\_\_\_

- 3** Find the midpoint of the segment with endpoints  $z=6-4i$  and  $w=-2+8i$ .

**ANSWER** \_\_\_\_\_

- 4** Find the midpoint of the segment with endpoints  $z=-5+i$  and  $w=3-7i$ .

**ANSWER** \_\_\_\_\_

- 5** Find the distance between  $z=2-5i$  and  $w=8+3i$ .

**ANSWER** \_\_\_\_\_

- 6** Find the midpoint of the segment with endpoints  $z=1/2+3i$  and  $w=7/2-i$ .

**ANSWER** \_\_\_\_\_

## APPLY IT Show your work

- 1** A game designer uses a complex plane for an in-game map. Two collectible items are at  $-3+4i$  and  $5-2i$ , where each unit is one map square. How many map squares apart are the items?

WORK SPACE

ANSWER \_\_\_\_\_

- 2** A video editor represents two animation keyframes as complex numbers  $-4-6i$  and  $8+2i$ . What complex number represents the midpoint between the keyframes?

WORK SPACE

ANSWER \_\_\_\_\_

## CHALLENGE One step up

### CHALLENGE

The endpoints of a segment are  $z=-2+7i$  and  $w=6-i$ . Find both the midpoint and the distance between the endpoints.

---

---

## PART 1 • PRACTICE

| # | PROBLEM                                                                    | ANSWER                    |
|---|----------------------------------------------------------------------------|---------------------------|
| 1 | Find the distance between $z=-2+3i$ and $w=4+3i$ .                         | <b>6</b>                  |
| 2 | Find the distance between $z=-1-2i$ and $w=2+2i$ .                         | <b>5</b>                  |
| 3 | Find the midpoint of the segment with endpoints $z=6-4i$ and $w=-2+8i$ .   | <b><math>2+2i</math></b>  |
| 4 | Find the midpoint of the segment with endpoints $z=-5+i$ and $w=3-7i$ .    | <b><math>-1-3i</math></b> |
| 5 | Find the distance between $z=2-5i$ and $w=8+3i$ .                          | <b>10</b>                 |
| 6 | Find the midpoint of the segment with endpoints $z=1/2+3i$ and $w=7/2-i$ . | <b><math>2+i</math></b>   |

## PART 2 • APPLY IT

- 10 map squares** Subtract the locations:  $(5-2i)-(-3+4i)=8-6i$ . The distance is  $|8-6i|=\sqrt{8^2+(-6)^2}=10$ .
- 2-2i** Average the two complex numbers:  $[(-4-6i)+(8+2i)]/2=(4-4i)/2=2-2i$ .

## CHALLENGE

**Midpoint:  $2+3i$ ; distance:  $8\sqrt{2}$**

Accept equivalent exact forms, including  $\sqrt{128}$  for  $8\sqrt{2}$ . Accept midpoints written with the real and imaginary parts in either order if the signs are correct.