

Beyond Real Roots

Use i to solve quadratics with negative discriminants.

HIGH SCHOOL • HSN-CN.C.7

CCSS.Math.Content.HSN-CN.C.7 "Solve quadratic equations with real coefficients that have complex solutions."

NAME _____

DATE _____

WHEN ROOTS TURN COMPLEX

For $ax^2 + bx + c = 0$, calculate the discriminant, $b^2 - 4ac$. If it is negative, use $\sqrt{-n} = i\sqrt{n}$ in the quadratic formula, and write the two conjugate complex solutions.

WORKED EXAMPLE

Solve $x^2 - 10x + 29 = 0$.

1. $a = 1$, $b = -10$, and $c = 29$, so $b^2 - 4ac = 100 - 116 = -16$.

2. $x = (10 \pm \sqrt{-16})/2$.

3. $x = (10 \pm 4i)/2 = 5 \pm 2i$.

Answer: $x = 5 + 2i$ or $x = 5 - 2i$

PRACTICE 6 problems

1 Solve $x^2 + 9 = 0$.

ANSWER _____

2 Solve $x^2 - 6x + 18 = 0$.

ANSWER _____

3 Solve $2x^2 + 4x + 10 = 0$.

ANSWER _____

4 Solve $(x - 5)^2 + 7 = 0$.

ANSWER _____

5 Solve $3x^2 - 6x + 11 = 0$.

ANSWER _____

6 Solve $4x^2 + 4x + 9 = 0$.

ANSWER _____

APPLY IT Show your work

- 1** A student council models the profit from a concert fundraiser by $P(x) = x^2 - 12x + 40$, where x is a real change in ticket price, in dollars. Find the values of x that make $P(x) = 0$. What does your result mean for real ticket prices?

WORK SPACE

ANSWER _____

- 2** A student is tuning a game controller. Its error score is modeled by $E(x) = 2x^2 + 8x + 17$, where x is a real sensitivity adjustment. Find the adjustments that would make $E(x) = 0$. What does your result mean?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Solve $12x^2 + 12x + 7 = 0$. Then, in one sentence, explain why neither solution is real.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Solve $x^2 + 9 = 0$.	$x = 3i$ or $x = -3i$
2	Solve $x^2 - 6x + 18 = 0$.	$x = 3 + 3i$ or $x = 3 - 3i$
3	Solve $2x^2 + 4x + 10 = 0$.	$x = -1 + 2i$ or $x = -1 - 2i$
4	Solve $(x - 5)^2 + 7 = 0$.	$x = 5 + \sqrt{7}i$ or $x = 5 - \sqrt{7}i$
5	Solve $3x^2 - 6x + 11 = 0$.	$x = 1 + (2\sqrt{6}/3)i$ or $x = 1 - (2\sqrt{6}/3)i$
6	Solve $4x^2 + 4x + 9 = 0$.	$x = -1/2 + \sqrt{2}i$ or $x = -1/2 - \sqrt{2}i$

PART 2 • APPLY IT

- $x = 6 + 2i$ or $x = 6 - 2i$. There is no real ticket-price change that makes the profit equal to 0. Set $x^2 - 12x + 40$ equal to 0. The discriminant is -16, so the roots are $6 \pm 2i$ and there are no real break-even ticket prices.
- $x = -2 + (3\sqrt{2}/2)i$ or $x = -2 - (3\sqrt{2}/2)i$. No real sensitivity adjustment makes the error score equal to 0. Set $2x^2 + 8x + 17$ equal to 0. The discriminant is -72, giving roots $-2 \pm (3\sqrt{2}/2)i$, so zero error is not reached for a real adjustment.

CHALLENGE

$x = -1/2 + (\sqrt{3}/3)i$ or $x = -1/2 - (\sqrt{3}/3)i$. Neither solution is real because the discriminant is -192, which is negative.

Accept either order for each pair of conjugate solutions and algebraically equivalent simplified forms. For the context problems, accept a clear statement that no real input makes the modeled quantity equal to zero.